INTERNATIONAL COLLEGE of YAYASAN MELAKA

ICYM

Open Distance Learning

LMAT 1013

MATHEMATICS 1

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UNDERSTANDING COURSE GUIDE

Refer and understand this *Course Guide* carefully from the beginning to the end. It describes the course and how you use the course material. It suggests the learning time to complete the course successfully. Referring the *Course Guide* will help you to clarify important contents that you might miss or overlook.

ABOUT THE COURSE

LMAT 1013 MATHEMATICS 1 is subject for Diploma Cyber Security that offered by School of Engineering and Computing Technology in ICYM. This course is worth 3 credit hours and should be covered to 14 weeks

You should be acquainted with learning independently and being able to optimize the learning modes and environment available to you. Make sure refer right course material and understand the course requirements as well as how the course is conducted.

LEARNING TIME SCHEDULE

It is a standard ICYM practice that learner accumulate 40 study hours for every credit hour. As for this three-credit hour course, you are expected to spend 120 study hours. Table 1 gives an estimation of how the 120 study hours could be accumulated.

Table 1: Estimation of Student Learning Time

Distribution of Student Learning Time by Chapter	CLO	Teaching and Learning Activities					
		Face to Face				Non-Face to Face (Independent Learning)	Total
		L	T	P	O		
Chapter 1	1		1			15.5	
Chapter 2	2		1			16	
Chapter 3	1		1			14	
Chapter 4	3		1			13	
Chapter 5	3		1			11	
Chapter 6	3		1			10	
Chapter 7	2		1			10	
						Sub-Total SLT	96.5

Continuous Assessment		%	Face to Face		Non-Face to Face (Independent Learning)	
			Physical	Online		
1	Quiz	10		3	2	
2	Test	30		2	2	
3	Assignment	5		2	2	
4	Participant	5		1	2	
					Sub-Total SLT	16

Final Assessment		%	Face to Face		Non-Face to Face (Independent Learning)	
			Physical	Online		
1	Final Examination	50	2.5		5	
					Sub-Total SLT	7.5
					GRAND-Total SLT	**120**

COURSE LEARNING OUTCOME

By the end of this course, you should be able to:

1. Explain the concept of operations for numbers in standard form and base number based on formula involving arithmetic and geometric progression **(C1, PLO5)**

2. Apply equation and inequalities in compound statements conjunction, disjunction, negation, conditional and bi-conditional, Boolean polynomial, tautology or contradiction and logically equivalent. **(C3, PLO1)**

3. Classify the operations of indices and logarithm, functions, composite and inverse functions and graph of functions **(C3,PLO2)**

COURSE SYNOPSIS

This course is divided into 10 topics. The synopsis for each topic can be listed as follows:

Topic 1 students will learns the significant figure in single number or decimal number, standard form and also number bases including converting and perform operation addition and also subtraction.

Topic 2 students will learns about Equation and Inequalities in Linear, Quadratic and Absolute value.

Topic 3 students are introduced to Arithmetic and Geometric Progression including the formula of terms and sum of terms

Topic 4 students are introduced to Laws of indices and Logarithms and solve the equation in indices and logarithms

Topic 5 students will learns describes the domain and range of function and solve the operation in function. Student will study how to solve the composite function and inverse function.

Topic 6 students will learns about Graph of Linear, Quadratic, Cubic and Reciprocal function including how to sketch graph and find the values of x and y.

Topic 7 students learn about introduction to logic including statements, compound statement, connective, truth table of negation, conjunction, conditional and biconditional.

LEARNING GUIDANCE

The learning guidance is important to understand before you go through this module. Understanding the learning guidance will help you to organize your study of this course in a more objective and effective way. Generally, learning guidance for each topic is as follows:

Learning Outcomes: This part is to measurable, observable, and specific statement that clearly indicates what you should know and be able to do because of learning in each chapter. By go through each topic, you can continuously gauge your understanding of the topic.

Self-Learning Material: To aid you in your subsequent learning and to report on what you have learned. The activities are in-text questions (ITO) and self-assessment questions (SAQ), assignment on each chapter of the material to monitor and develop your own learning.

Self Assessment: Self-assessment question is such a task that requires written answer form you. If you complete the task, you are asking to check your answer with the answer key provided in the module.
Self -assessment is be developed in various form of test questions, there are easy question, fill in the blank, multiple choices, true-false and matching.

Summary: You will find this part at the end of each topic. This component helps you to recap the whole topic. By going through the summary, you should be able to gauge your knowledge retention level. Should you find points in the summary that you do not fully understand, it would be a good idea for you to revisit the details in the module.

Key Terms: This component can be found at the end of each topic. You should go through this component to remind yourself of important terms or jargon used throughout the module. Should you find terms here that you are not able to explain, you should look for the terms in the module.

References: The References section is where a list of relevant and useful textbooks, journals, articles, electronic contents, or sources can be found. The list can appear in a few locations such as in the *Course Guide* (at the References section), at the end of every topic or at the back of the module. You are encouraged to read or refer to the suggested sources to obtain the additional information needed and to enhance your overall understanding of the course.

ASSESSMENT METHOD

Please refer to ICYM E Learning

TOPIC 1

NUMBER SYSTEM

LEARNING OUTCOMES

By the end of topic, you should be able to:

1. Explain the decimal places, significant figures and standard form.
2. Identify decimal, binary, octal and hexadecimal number system.
3. Calculate the conversion of different bases to decimal and vice verse.

NUMBER

Numerical 'digits' or 'numerals' are referred to as 'numbers.' Digits are distinct symbols or letters (such as '0, 1', '3', or '7) that are used alone or in groups to denote a number (such as '37' or '1073'). Numbers are used in mathematics to count, measure, and compute. Number can be in whole number or single number, decimal or fraction number.

The term "integer" refers to a "whole" number that can be written without the use of a decimal point or a fraction. Positive and negative integers are both possible. All of the numbers are integers: 1, 7, 375, 56, 12, and 8.

Different number bases are used in different number systems. When employing a certain numbering system, a number base tells how many different digits are available.

1.1 The significant figures, and standard form

A. Significant Form

Significant figures are meaningful digits in a measured quantity.

Rules for rounding off a positive number to a specific significant figure:

Any digit that is not zero is significant figure.

Example:

i. 64 798	– 5 significant figures
ii. 7.98	– 3 significant figures

Zero between non zero digit is significant figure

Example:

i. 6 034	– 4 significant figures
ii. 60 045	– 5 significant figures
iii. 60.02	– 4 significant figures

Zero to the right of non-zero digit is not significant except stated.

Example:

i. 40	– 1 significant figures
ii. 30 70	– 3 significant figures
iii. 73 550 = 74 000	– 2 significant figures
iv. 73 550 = 73 600	– 3 significant figures

Zero to the right of non-zero digit (in a decimal number) is significant figure.

Example:

i. 0.80	– 2 significant figures
ii. 7.980	– 4 significant figures

Zero to the left of non-zero digit (in a decimal) number is not significant figures.

Example:

i. 0.080	– 2 significant figures
ii. 0.00708	– 3 significant figures

Example 1:

State the significant figure of the number given:

a. 0.0401
b. 73 640
c. 30.470

SOLUTION:

a. 3 significant figure
b. 4 significant figure
c. 5 significant figure

Example 2:

State the significant figure of the number given:

a. 0.0401
b. 73 640
c. 30.470

SOLUTION:

a. 1 significant figure
b. 3 significant figure
c. 2 significant figure

B. Round off

How to perform round off ?

- Decide which is significant figure first.
- Next to significant figure (right side) is Rounder decider.
- If the decider is 5 or above, increase the previous value by 1. If not, its unchanged.

After round off?

If the number is **single number**, replace zero after round off.

If the number is **decimal number**, no need to replce zero after round off.

Example 3:

Round off the whole number and decimal number to the number 1, 2 and 3 significant figure.

a. 20671

b. 0.007321

c. 38.063

d. 0.09994

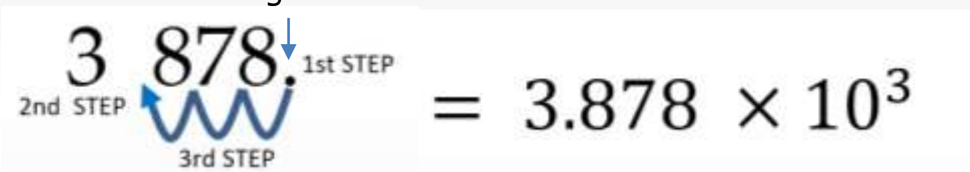
SOLUTION:

Number	1 significant figure	2significant figures	2 significant figures
20 671	20 000	21 000	20 700
0.007321	0.007	0.0073	0.00732
38.063	40	38	38.1
0.08994	0.09	0.090	0.0899

C. Standard form

$$A \times 10^n, where\ 1 \leq x < 10\ and\ n\ is\ an\ integers$$

How to convert from single number to standard form?



1. Determine where is decimal point (if there is no decimal point stated, the decimal point should be behind the last right number)

2. Move the decimal point until there is only one non-zero digit to the left of the decimal point.

3. Count how many places you moved the decimal point to determine the

value of n

- If you moved the decimal to the left, the **value of n is positive.**
- If you moved the decimal to the right, the **value of n is negative.**

Example 4:

Express each of the following number in the standard form correct to 3 significant figures.

a. 371.1
b. 0.008356
c. 7450
d. 909.5×10^2

SOLUTION:

a. $3.711 \times 10^2 = 3.71 \times 10^2$ [$3s.f$]
b. $8.356 \times 10^{-5} = 8.37 \times 10^{-5}$ [$3s.f$]
c. $7.450 \times 10^3 = 7.45 \times 10^3$[$3s.f$]
d. $9.095 \times 10^4 = 9.10 \times 10^4$[$3s.f$]

How to convert from standard form to single number

$$3.07 \times 10^{-3} = 0.00307$$

1. Determine where is decimal point and the value of n

2. If the value of n is positive, move the decimal point to the right
   If the value of n is negative, move the decimal point to the left according to the value to n.

Example 5:

State the given standard form as a single number correct to 3 significant figures.

a. 8.7590×10^2
b. 6.734×10^{-3}

SOLUTION:

a. $8.7590 \times 10^2 = 875.90 = 876$[$3s.f$]
b. $6.734 \times 10^{-3} = 0.006734 = 0.00673$ [$3s.f$]

D. Operation +, –, × and ÷ according to BODMAS

B O D M A S

$$(x) \quad x^2 \quad \div \quad \times \quad + \quad -$$

BODMAS have been used to in order of operations to be followed while solving expressions in mathematics. The acronym stands for **B - Brackets, O - Order of powers, D - Division, M - Multiplication, A - Addition,** and **S - Subtraction.**

Example 6:

Calculate each of the following and state the answer in standard form correct to 3 significant figures.

$$3650 \div 2 + (8.2 \times 10^2 + 5.6) =$$

SOLUTION:

$= 3650 \div 2 + (820 + 5.6)$
$= 3650 \div 2 + 825.6$
$= 1825 + 825.6$
$= 2650.6$
$= 2.65 \times 10^3$

Explanation: Follow the BODMAS rule.

1st step: Solve in the bracket first. Do the order first after that addition operation.

2nd step: Solve the division then the addition operation

3rd step: convert the single number to standard form and 3 significant figure

Example 7:

Calculate the number and state the answer in standard form correct to 3 significant figures.

i. $2 \times 0.2 - \frac{(519\times10^{-2})}{89}$

ii. $\left(\frac{118.8+4.23}{7.6\times10^{2}}\right)^{2}$

SOLUTION:

i) $= 0.4 - \frac{5.19}{89}$
$= 0.4 - 0.058315$
$= 0.3941685$
$= 3.94 \times 10^{-1}$

ii) $=\left(\frac{123.03}{760}\right)^{2}$
$= 0.0262056456$
$= 2.62 \times 10^{-2}$

1.2 BINARY, OCTAL AND HEXADECIMAL NUMBER SYSTEM

A. Number Base

The number base determines how many digits are needed to represent a number. Table 1.1 explained the number that involve in decimal, binary, octal and hexadecimal number system

Name	Base	Digit that involve	Exampl e
DECIMAL NUMBER	BASE 10	0 1 2 3 4 5 6 7 8 9	56_{10}
BINARY NUMBER	BASE 2	0 1	1011_{2}
OCTAL NUMBER	BASE 8	0 1 2 3 4 5 6 7	546_{8}

HEXADECIMAL NUMBER	BASE 16	0 1 2 3 4 5 6 7 8 9 A as (10) B as (11) C as (12) D as (13) E as (14) F as (15)	$BE6_{16}$

Table 1.1: The number that involve in decimal, binary, octal and hexadecimal number system

B. PLACE VALUE IN BINARY, OCTAL AND HEXADECIMAL

Numbers that cannot be represented by a single digit are arranged in columns called place values. The place values in the following examples are shown as labelled boxes for each column.

Example 8

Write down the place value of the base given.

a. 1011_2

b. 546_8

c. $BE6_{16}$

SOLUTION:

a. 1011_2

1	0	1	1
2^3	2^2	2^1	2^0

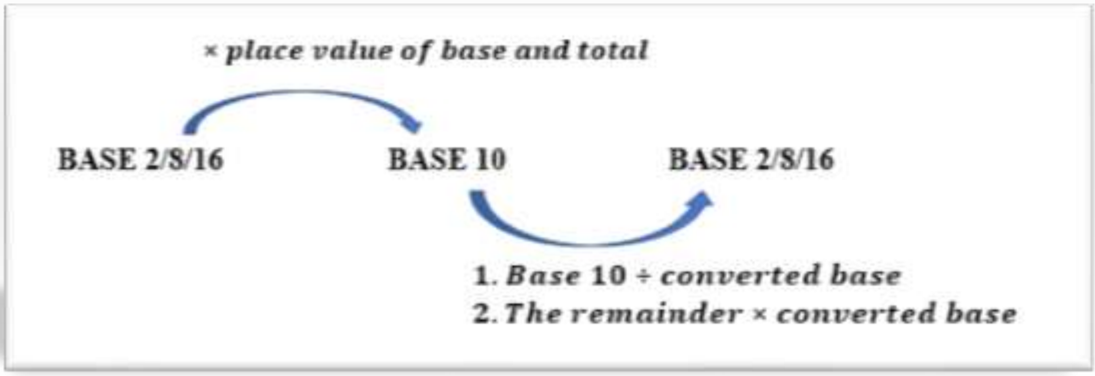
b. 546_8

5	4	6
8^2	8^1	8^0

c. $BE6_{16}$

B =11	E =14	6
16^2	16^1	16^0

C. CONVERT IN BASE BINARY/OCTAL/ HEXADECIMAL



Example 9

Convert $4\square2_{16}$ to the binary number system.

SOLUTION:

$A = 10$

$= (2\times 16^0) + (10 \times 16^1) + (4 \times 16^2)$

$= 2 + 160 + 1024$

$= 1186_{10}$

2	1186	
2	593	= 0
2	296	0.5 × 2 = 1
2	148	= 0
2	74	= 0
2	37	= 0
2	18	0.5 × 2 = 1
2	9	= 0
2	4	0.5 × 2 = 1
2	2	= 0
2	1	= 0
	0	0.5 × 2 = 1

$=10010100010_2$

Example 10

Convert 357_8 to the hexadecimal number system.

SOLUTION:

= $(3\times 8^0) + (5 \times 8^1) + (3 \times 8^2)$

= 3 + 40 + 192

= 235_{10}

16	235	
16	14	$0.6875 \times 16 = 11 = B$
16	0	$0.875 \times 16 = 14 = E$

= BE_{16}

Example 11

Convert 101101_2 to the octal number system.

SOLUTION:

= $(1 \times 2^0) + (0 \times 2^1) + (1 \times 2^2) + (1 \times 2^3) + (0 \times 2^4) + (1 \times 2^5)$

= 1 + 0 + 4 + 8 + 0 + 32

= 45_{10}

8	45	
8	5	$0.625 \times 8 = 5$
8	0	$0.625 \times 8 = 5$

= 55_8

1.3 ADDITION AND SUBTRACTION IN BINARY, OCTAL, AND HEXADECIMAL

In table 1.2 explained the convertion in decimal, binary, octal and hexadecimal number. With the help of this table addition of number base is best illustrated by the following examples.

Base 10	Base 2	Base 8	Base 16
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	6	5
6	110	7	6
7	111	10	7
8	1000	11	8
9	1001	12	9
10	1010	13	10 = A
12	1011	14	11 = B

Table 1.2 : the convertion in decimal, binary, octal and hexadecimal number.

Example 11

$1011_2 + 1111_2 =$
$1001_2 - 110_2 =$

SOLUTION:

$$\begin{array}{r} \color{red}{111} \\ 1011_2 \\ +\ 1111_2 \\ \hline =11010_2 \end{array}$$

$$\begin{array}{r} \color{red}{12} \\ \not{1}001_2 \\ -\ \ 110_2 \\ \hline 11_2 \end{array}$$

=

Example 12

$402_8 + 643_8 =$
$624_8 - 74_8 =$

SOLUTION:

$$\begin{array}{r} 402_8 \\ +\ 643_8 \\ \hline =1245_8 \end{array}$$

$$\begin{array}{r} {\scriptstyle 5\ 12} \\ \not{6}\not{2}4_8 \\ -\quad 74_8 \\ \hline =530_8 \end{array}$$

Self Assessment

Exercise 1.1

1. Write the following numbers in standard form and correct to 2 significant figures.

a. 0.00517
b. 534.900
c. 82

2. Write the following as single number and correct to 3 significant figures.

a. 3.42×10^3
b. 0.3256×10^{-4}

3. Find each of the following values to 3 significant values and state your answer in the standard form

a. $\frac{45888}{222\times10^3} + 444$

$b. \frac{(1.0432\times3.01521)}{5.004}$

Exercise 1.2

A. Convert each of the following number to base that have been stated.

1. 1101110_2

a. $Base_{10}$
b. $Base_8$
c. $Base_{16}$

2. 176_8

a. $Base_{10}$
b. $Base_2$
c. $Base_{16}$

B. Solve the binary, octal problems below

a. $111001_2 - 11010_2$
b. $11101_2 + 111_2$
c. $46_8 + 3125_8$
d. $705_8 - 611_8$

Answer

EXERCISE 1.1

1. a. 5.2×10^{-3} b. 5.3×10^{2} c. 8.2×10^{1}
2. a. 3420 b. 0.0000326
3. a. 4.44×10^{2} b. 6.29×10^{-1}

EXERCISE 1.2 & 2.3

A.

1. $a.$ 110_2 $b.$ 156_8 $c.$ $6E_{16}$
2. $a.$ 126_{10} $b.$ 1111110_2
   $c.$ $7E$

B.

a. 11111_2

b. 100100_2

c. 3173_8

d. 74_8

KEY TERM

Number system | **Standard form**

Significant figure | **Number base**

SUMMARY

- Number system involve decimal places, significant figures and standard form.
- Number base including decimal, binary, octal and hexadecimal base
- conversion of different bases to decimal by multiplying with place value and converting decimal to different bases by division of the converting base.

REFERENCEES

1. Ong, B. S., Zubairi, Y. Z., & Lee, K. Y. (2010b).Number System. *Mathematics for Matriculation*. Oxford University Press. 1-29

2. Fahad, E. (2021, April 4). Number System in Computer, Binary, Octal, Decimal, Hexadecimal. Electronic Clinic. *https://www.electroniclinic.com/number-system-in-computer-binary-octal-decimal-hexadecimal/*

TOPIC 2

Equation and inequalities

LEARNING OUTCOMES

By the end of topic, you should be able to:

1. Explain the equations and inequalities of functions.
2. Solving equations and inequalities of linear, quadratic, absolute value and rational functions.

INTRODUCTION

A mathematical statement that claims the equivalence of two expressions is known as an equation. The statement "three plus two equals five" is expressed by the equation 3+2=5, for example.

An equation frequently contains one or more variables. Each expression must still be written on either side of an equals sign (=). For instance, the equation x+1=4, which means "x plus one equals four,".

It's not always about "equals" in mathematics; sometimes all we know is that something is more or less than. Thus, inequalities have been use to tells us about the relative size of values. Integers, variables, and several other algebraic expressions can all be compared with them. There are 4 type of relations which are $>, <, \geq,$ and $\leq$

To solve an inequality by changing it into a symbol with a variable on one side and a number or expression on the other. Thus, numerous operations are frequently required to change an inequity in this way such as addition, subtraction, multiplication and division.

2.1 Equation of Function

TYPE OF EQUATION

- **Linear equation** – an equation which contain one or more unknowns of the first degree. The standard form for linear equation $\boldsymbol{mx + b = 0}$ where m and b are numbers and x is a variable. Example :

$$5x + 3 = 0$$

$$2x + 3y = 6$$

- **Quadratic equation**- an equation which contain variable in second degree (the highest degree). The standard form for linear equation $\boldsymbol{ax^2 + bx + c = 0}$ where a, b and c are numbers and x is a variable. A quadratic equation is of degree 2, thus it has 2 solutions or roots

$$2x^2 + 3x = 7$$

$$4y^2 + 2y - 16 = 7$$

- **Absolute value equation** , |x| - Absolute value describes the distance from zero that a number is on the number line, without considering direction. Has two solutions x = a and x = -a because both numbers are at the distance a from 0.

$$\left|\frac{1}{2}x + 4\right| = 6$$

thus,

$$\left|\frac{1}{2}x + 4\right| = \begin{cases} 6 & if\ x \geq 0 \\ -6 & if\ x < 0 \end{cases}$$

A. i) LINEAR EQUATION WITH ONE VARIABLE

Example 1 i

Solve the following linear equation:

i) $3x - 8 = 7$

ii) $5x - 9 = 3x + 7$

iii) $w(w - 3) = w^2 + 9$

iv) $\frac{x-4}{-3} = \frac{x-2}{5}$

v) $1 - 3(x - 3) = 2x + 5$

vi) $3x + 5 = 6 - 2(x - 2)$

SOLUTION:

i) $3x - 8 = 7$

$3x = 7 + 8$

$3x = 15$

$x = \frac{15}{3}$

$x = 5$

ii) $5x - 9 = 3x + 7$

$5x - 3x = 7 + 9$

$2x = 16$

$x = \frac{16}{2}$

$x = 8$

iii) $w(w - 3) = w^2 + 9$

$w^2 - 3w = w^2 + 9$

$w^2 - w^2 - 3w = +9$

$-3w = +9$

$w = \frac{9}{-3} x = -3$

iv) $\frac{x-4}{-3} = \frac{x-2}{5}$

$5(x - 4) = -3(x - 2)$

$5x - 20 = -3x + 6$

$5x + 3x = 6 + 20$

$8x = 26$

$x = \frac{26}{8} = \frac{13}{4}$

v) $1 - 3(x - 3) = 2x + 5$

$1 - 3x + 9 = 2x + 5$

$-3x + 10 = 2x + 5$

$-3x - 2x = 5 - 10$

$-5x = -5$

$x = \frac{-5}{-5} = 1$

vi) $3x + 5 = 6 - 2(x - 2)$

$3x + 5 = 6 - 2x + 4$

$3x + 5 = 10 - 2x$

$3x + 2x = 10 - 5$

$5x = 5$

$x = 1$

Example 1 ii

Solve the following linear equation:

i) $\frac{3x}{3} = \frac{3x+4}{2}$

ii) $6 - 4(3 - x) = 5x - 2$

iii) $\frac{2-2(x-3)}{-4} = x + 3$

SOLUTION:

i. $\frac{3x}{3} = \frac{3x+4}{2}$

$$2(3x) = 3(3x + 4)$$
$$6x = 9x + 12$$
$$6x - 9x = 12$$
$$-3x = 12$$
$$x = \frac{12}{-3}$$
$$x = -4$$

ii. $6 - 4(3 - x) = 5x - 2$

$$6 - 12 + 4x = 5x - 2$$
$$-6 + 4x = 5x - 2$$
$$4x - 5x = -2 + 6$$
$$-x = 4$$
$$x = \frac{4}{-1}$$
$$x = -4$$

iii. $\frac{2-2(x-3)}{-4} = x + 3$

$$2 - 2(x - 3) = -4(x + 3)$$
$$2 - 2x + 6 = -4x - 12$$
$$8 - 2x = -4x - 12$$
$$-2x + 4x = -12 - 8$$
$$2x = -20$$
$$x = -\frac{20}{2}$$
$$x = -10$$

ii) LINEAR EQUATION WITH TWO VARIABLE

There are two methods in solving a system of linear equation.

- Substitution
- Elimination

Example 2

Solve the following system linear equation:

$$3x - y = 7$$
$$2x + 3y = 1$$

SOLUTION:

By using substitution methods:

$$3x - y = 7 \ldots Eq\ 1$$
$$2x + 3y = 1 \ldots Eq\ 2$$

$3x - y = 7$
$3x - 7 = y \ldots Eq\ 3$

Insert Eq 3 into Eq 2

$2x + 3(3x - 7) = 1$
$2x + 9x - 21 = 1$
$11x - 21 = 1$
$11x = 22$
$x = \frac{22}{11}$
$x = 2$

Insert the value of x into eq 1

$3(2) - y = 7$
$6 - y = 7$
$y = 6 - 7$
$y = -1$

By using elimination methods:

$$3x - y = 7 \ldots Eq\ 1$$
$$2x + 3y = 1 \ldots Eq\ 2$$

$3(3x - y = 7)$
$2x + 3y = 1$

$$9x - 3y = 21$$
$$(+)\quad 2x + 3y = 1$$
$$11x = 22$$
$$x = \frac{22}{11}$$
$$x = 2$$

Insert the value of x into eq 1

$3(2) - y = 7$
$6 - y = 7$
$y = 6 - 7$
$y = -1$

III. QUADRATIC EQUATION.

General form of Quadratic equation

$$ax^2 + bx + c = 0$$

where $a \neq 0, b$ *and* c *is constant.*

Methods of solving Quadratic Equations

- Factorization
- By using formula

i) Factorization

Example 3 i

Solve the following quadratic equation:

- $x^2 + 2x - 15 = 0$
- $x^2 - 16 = 0$
- $2x^2 + x = +6$

SOLUTION:

i. $\mathbf{x^2 + 5x - 4 = 0}$

find a, b, c

$a = 1$
$b = 5$
$c = -4$

By using calculator (EQN):

$(x - 3)(x + 5) = 0$
$x = +3 \quad x = -5$

ii. $\mathbf{b)}$ $\mathbf{x^2 - 16 = 0}$

$x^2 = 16$
$x = \sqrt{16}$
$x = \pm 4$
$x = +4 \; x = -4$

How to find the values of x using calculator using EQN?

1. *Press MODE 3 times. Choose EQN.*
2. *Choose degree of 2*
3. *Insert the value of a, b, and c*

iii. $\mathbf{2x^2 + x = +6}$

$\mathbf{2x^2 + x - 6 = 0}$ → *the right side must be 0*

find a, b, c

$a = 2$
$b = 1$
$c = -6$

By using calculator (EQN):

$(x + 2)(2x - 3) = 0$

$x = -2 \quad x = \frac{3}{2}$

Example 3 ii

Solve the following quadratic equation:

- $(a-1)(a-5) = 1-a$
- $\frac{(x+1)(x+7)}{8} = 1$
- $x+3 = \frac{3}{x-1}$

SOLUTION:

i. $(a-1)(a-5) = 1-a$

$(a^2-5a-a+5) = 1-a$

$(a^2-6a+a+5-1) = 0$

$a^2-5a+4 = 0$

find a, b, c

$a = 1$

$b = -5$

$c = +4$

By using calculator (EQN):

$(x-1)(x-5) = 0$

$x = +1 \quad x = 4$

ii. $\frac{(x+1)(x+8)}{4} = 2$

$(x+1)(x+8) = 8$

$x^2+8x+x+8-8 = 0$

$x^2+9x = 0$

find a, b, c

$a = 1$

$b = 9$

$c = 0$

By using calculator (EQN):

$(x+9)(x) = 0$

$x = -9 \; x = 0$

iii. $x+3 = \frac{3}{x-1}$

$(-x+3)(x-1) = 5$

$-x^2+x+3x-3 = 3$

$-x^2+4x-3-3 = 0$

$-x^2+4x-8 = 0$

find a, b, c

$a = -1$

$b = +4$

$c = 6$

By using calculator (EQN):

$(x-5.162)(x+1.162) = 0$

$x = 5.162 \quad x = -1.162$

ii) by using formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 4

Solve those equations by using formula:

a) $5x^2 + 9x - 2 = 0$

b) $(x-2)(x-1) = -4$ by using formula in 3 significant figure.

SOLUTION:

Find the value of a, b and c

$a = 5 \;\; b = 9 \;\; c = -2$

by using formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(9) \pm \sqrt{(9)^2 - 4(5)(-2)}}{2(5)}$$

$$x = \frac{-(9) \pm \sqrt{121}}{10}$$

$$x = \frac{-(9) + \sqrt{121}}{10}$$

$$x = \frac{2}{10}$$

$$x = \frac{1}{5}$$

$$x = \frac{-(9) - \sqrt{121}}{10}$$

$$x = -\frac{20}{10}$$

$$x = -2$$

Solve:

$(x-2)(x-1) = 3$

$x^2 - 2x - x + 2 - 3 = 0$

$x^2 - 3x - 1 = 0$

Find the value of a, b and c

$a = 1 \;\; b = -3 \;\; c = -1$

by using formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{(3) \pm \sqrt{1}}{2}$$

$$x = \frac{3 + \sqrt{13}}{2}$$

$x = 3.30$

$$x = \frac{3 - \sqrt{13}}{2}$$

$x = -0.303$

C. ABSOLUTE VALUE

Example 5

Solve the equations below

a) $|\frac{1}{2}\mathrm{x}+4| = 6$

b) $\left|\frac{4}{3x-2}\right| = 3$

SOLUTION:

$a)$ $|\frac{1}{2}\mathrm{x}+4| = 6$

$$\frac{1}{2}\mathrm{x}+4 = 6$$
$$\frac{1}{2}\mathrm{x} = 6-4$$
$$x = 2(2)$$
$$x = 4$$

$$\frac{1}{2}\mathrm{x}+4 = -6$$
$$\frac{1}{2}\mathrm{x} = -6-4$$
$$x = -10(2)$$
$$x = -20$$

b) $\left|\frac{4}{3x-2}\right| = 3$

$$\frac{4}{3x-2} = 3$$
$$4 = 3(3x-2)$$
$$4 = 9x-6$$
$$4+6 = 9x$$
$$10 = 9x$$
$$\frac{10}{9} = x$$

$$\frac{4}{3x-2} = -3$$
$$4 = -3(3x-2)$$
$$4 = -9x+6$$
$$4-6 = -9x$$
$$-2 = -9x$$
$$\frac{2}{9} = x$$

Example 6

Solve the equations below

$a)\ |1 - x| = 2x - 5$

$b)\ |x^2 + 4x - 4| = 2$

SOLUTION:

$a)\ |1 - x| = 2x - 5$

$$1 - \mathrm{x} = 2x - 5$$
$$3\mathrm{x} = 6$$
$$\mathrm{x} = \frac{6}{3}$$
$$\mathrm{x} = 2$$

$$1 - \mathrm{x} = -(2x - 5)$$
$$1 - x = -2\mathrm{x} + 5$$
$$x = 4$$

b) $|x^2 + 5x - 5| = 1$

$$x^2 + 5x - 5 = 1$$
$$x^2 + 5x - 5 - 1 = 0$$
$$x^2 + 5x - 6 = 0$$
$$(x - 1)(x + 6) = 0$$
$$x = 1 \quad x = -6$$

$$x^2 + 5x - 5 = -1$$
$$x^2 + 5x - 5 + 1 = 0$$
$$x^2 + 5x - 4 = 0$$
$$(x - 0.702)(x + 5.702) = 0$$
$$x = 0.702 \quad x = -5.702$$

2.2 Inequalities of Function

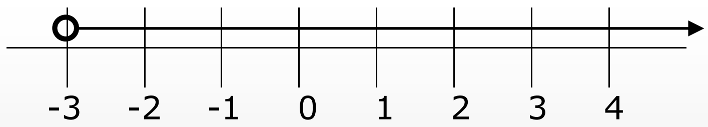
Inequality involves one of the four symbols.

$$>, \geq, < or \;\leq$$

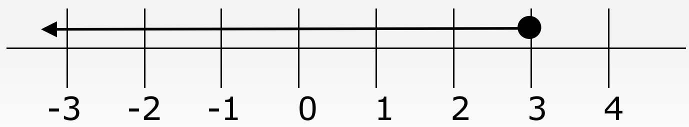
Inequalities can be represented on a number line as shown in the following worked examples. Use an open circle for > and < while a closed circle for ≥ and ≤.

Example:

The graph for $x > -3$



The graph for $x \leq 3$



The properties of inequalities are as follow:

If a>b, then

1. $a + c > b + c, c > 0$
2. $ac > bc, c > 0$
3. $ac < bc, c > 0$
4. $\frac{1}{a} < \frac{1}{b}, whre\ a, b \neq 0$

TYPE OF INEQUALITIES

- **Linear inequalities** – an inequalities which contain one variable of the first degree. The standard form for linear inequalities can be written in one of the following forms:

$$ax + b < 0 \qquad ax + b \leq 0$$
$$ax + b > 0 \qquad ax + b \geq 0$$

where x is a variable, a is a non zero real number and b is a real number. Example:

$$5x + 3 > 0$$

Quadratic inequalities- an inequalities which contain unknown in second degree. The standard form for quadratic inequalities can be written in one of the following forms:

$$ax^2 + bx + c < 0$$
$$ax^2 + bx + c \leq 0$$
$$ax^2 + bx + c > 0$$
$$ax^2 + bx + c \geq 0$$

where a, b and c are numbers and x is a variable. A cannot be 0 ($a \neq 0$). A quadratic equation is of degree 2, thus it has 2 solutions or roots

$$2x^2 + 3x \leq 7$$

- **Absolute value inequalities** , |x| - The phrase |x|=a can be taken to signify that the distance between x and the origin is smaller than a; as a result, x must fall on the real number line between -a and a. In other words: $|x| < a \rightarrow -a < x < a$

  similarly:

  - $|x| \leq a \rightarrow -a \leq x \leq a$
  - $|x| > a \rightarrow x > a \ or \ x < -3$
  - $|x| \geq a \rightarrow x \leq a \ or \ x \leq -3$
  -

  For example:

$$\left|\frac{1}{2}x + 4\right| < 6$$

A. LINEAR INEQUALITIES WITH ONE VARIABLE

Example 7

Solve the following inequalities:

$a)\ 2x - 10 > 8$

$b)\ 2 + x \geq 4 + 5x$

$c)\ \dfrac{2(x-7)}{3-x} < 3$

$d)\ 3(x-1) \leq 4(x+2)$

$e)\ -6 < x - 5 \leq 1$

$f)\ 6x \leq x - 10 < 3x - 8$

SOLUTION:

a) $2x - 10 > 8$

$2x > 8 + 10$

$2x > 18$

$x > \dfrac{18}{2}$

$x > 9$

b) $2 + x \geq 4 + 5x$

$x - 5x \geq 4 - 2$

$-4x \geq 2$

$x \leq \dfrac{2}{-4}$

$x \leq \dfrac{1}{-2}$

Multiplying or dividing both sides by a negative number reverses the inequality.

This means < changes to >, and vice versa.

c) $\frac{2(x-7)}{3-x} < 3$

$2(x-7) < 3(3-x)$

$2x - 14 < 9 - 3x$

$2x + 3x < 9 + 14$

$5x < 23$

$x < \frac{23}{5}$

d) $3(x-1) \leq 4(x+2)$

$3x - 3 \leq 4x + 8$

$3x - 4x \leq 8 + 3$

$-x \leq 11$

$x \geq -11$

e) $-6 < x - 5 \leq 1$

$-6 < x - 5$

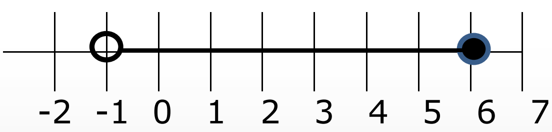
$-6 + 5 < x$

$-1 < x$

$x - 5 \leq 1$

$x \leq 1 + 5$

$x \leq 6$



f) $6x \leq x - 10 < 3x - 8$

$6x \leq x - 10$

$6x - x \leq -10$

$5x \leq -10$

$x \leq -\dfrac{10}{5}$

$x \leq -5$

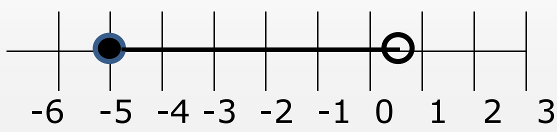
$x - 10 < 3x - 8$

$x + 3x < -8 + 10$

$4x < 2$

$x < \dfrac{2}{4}$

$x < \dfrac{1}{2}$



IV.

QUADRATIC EQUATION.

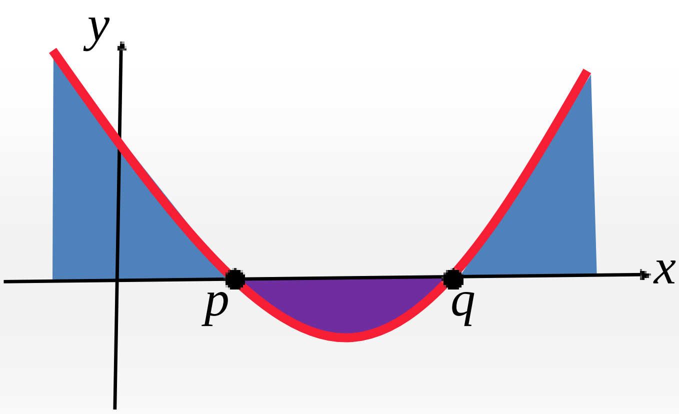
Find the quadratic inequalities by algebraic or graphical approach.

Quadratic inequalities can be solved by using algebraic method and graphical method.

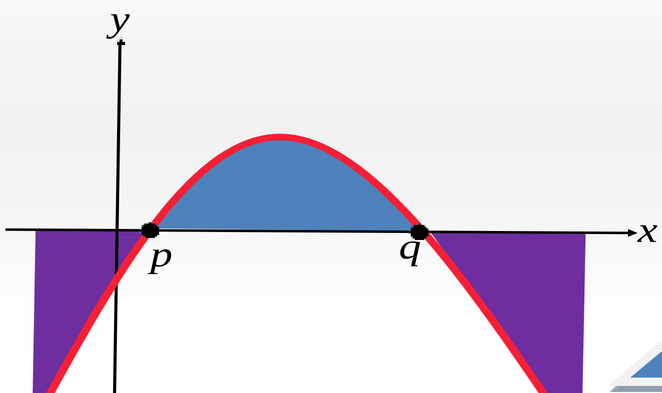
Graphical method:

The graph of quadratic expression y = ax2 + bx + c is sketched and points where the graph cuts the x-axis, say p and q are noted.

For a > 0
y > 0 when x < p or x > q
y < 0 when p < x < q



for a < 0
y > 0 when p < x < q
y < 0 when x < p or x > q



Example 8

Solve the following inequalities by using graphical method.

$a)\ x^2 + 5x + 6 > 0$

$b)\ 2 - 5x - 3x^2 < 0$

SOLUTION:

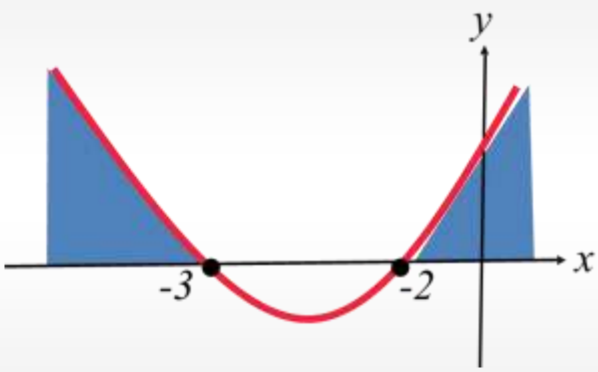
a) $x^2 + 5x + 6 > 0$

$(x + 2)(x + 3) > 0$

for a > 0

y > 0 when x < p or x > q

Thus, x<-3 or x>-2



b) $2 - 5x - 3x^2 > 0$

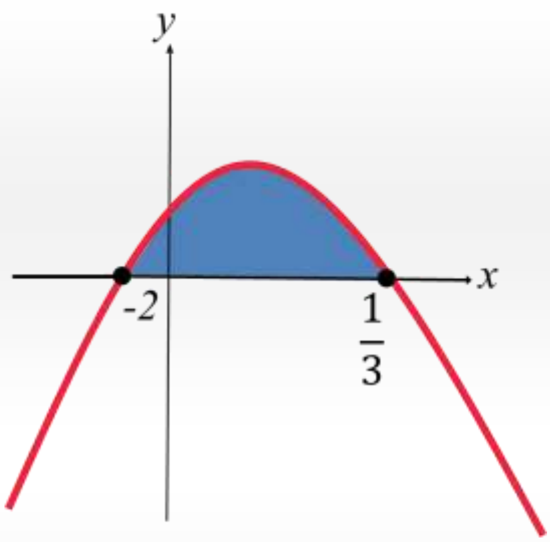
$(3x - 1)(x + 2) > 0$

$x = \frac{1}{3} \qquad x = -2$

for a < 0

y > 0 when p < x < q

Thus , $-2<x<\frac{1}{3}$



Algebraic method

1) Use the properties of inequalities to obtain a zero on one side of the inequality.
2) Factorize, if possible, the quadratic expression on the left hand side.
3) Determine the critical values of x. (A value for which a factor is zero is called a critical value of x.)
4) Construct the table with interval and factors
5) Choose a suitable value for each interval and evaluate each factor to determine the sign.

Example 9

Solve the following quadratic inequalities by using algebraic method

$a)\ x^2 - 2x - 8 \geq 0$

b)$3 - 5x < -2x^2$

SOLUTION:

a) $x^2 - 2x - 8 \geq 0$

$(x-4)(x+2) \geq 0$

The value of x	Less than -2	$-2 < x < 4$	More than 4
x	-3	0	5
$(x-4)$	$-$	$-$	$+$
$(x+2)$	$-$	$+$	$+$
$(x-4)(x+2)$	$+$	$-$	$+$

$We\ choose\ +\ results\ as\ y \geq 0 /\ Pilih\ +\ disebabkan\ y \geq 0$

Thus, $x \leq -2\ or\ x \geq 4$

$b)\ 3 - 5x < -2x^2$

$3 - 5x + 2x^2 < 0$

$(2x-3)(x-1) < 0$

The value of x	Less than 1	$1 < x < \frac{3}{2}$	More than $\frac{3}{2}$
x	0	1.3	2
$(2x-3)$	$-$	$-$	$+$
$(x-1)$	$-$	$+$	$+$
$(2x-3)(x-1)$	$+$	$-$	$+$

$We\ choose - \ results\ as\ y < \ 0/\ Pilih - \ disebabkan\ y < \ 0$

Thus, $1 < x < \frac{3}{2}$

V. RATIONAL INEQUALITIES

Inequalities of the form $\frac{p(x)}{Q(x)} < 0$, where is linear expression are easier solved by algebraic method.

Example 10

Solve the following inequalities by using algebraic method

$a) \frac{x-5}{1-x} \geq 1$

SOLUTION:

$$\frac{x-5}{1-x} \geq 1$$
$$\frac{x-5}{1-x} - 1 \geq 0$$
$$\frac{x-5}{1-x} - \frac{1-x}{1-x} \geq 0$$
$$\frac{x-5-1+x}{1-x} \geq 0$$
$$\frac{2x-6}{1-x} \geq 0$$

$2x = 6$ $\quad\quad$ $-x = -1$

$= \frac{6}{2} = 3$ $\quad\quad$ $x = 1$

The value of x	Less than 1	$1 < x < 3$	More than 3
x	0	2	4
$(2x-6)$	$-$	$-$	$+$
$(1-x)$	$+$	$-$	$-$
$\frac{(2x-6)}{(1-x)}$	$-$	$+$	$-$

$We\ choose\ +\ results\ as\ y \geq 0$

Thus, $1 < x < 3$

D. ABSOLUTE VALUE INEQUALITIES

Example 11

Solve the equations below

c) $\left|\frac{1}{2}\mathrm{x}+4\right|>6$

d) $\left|\frac{4}{3x-2}\right|\leq 3$

SOLUTION:

$a)\ \left|\frac{1}{2}\mathrm{x}+4\right|>6$

$\frac{1}{2}\mathrm{x}+4>6$ OR $\frac{1}{2}\mathrm{x}+4<-6$

$\frac{1}{2}\mathrm{x}>6-4$ $\qquad$ $\frac{1}{2}\mathrm{x}<-6-4$

$x>2(2)$ $\qquad$ $x<-10(2)$

$x>4$ $\qquad$ $x<-20$

Thus, $x<-20\ or\ x>4$

I. $\left|\frac{4}{3x-2}\right|\leq 3$

$$-3\leq\frac{4}{3x-2}\leq 3$$

$$-3(3x-2)\leq 4$$

$$\leq 3(3x-2)$$

$$-9x+6\leq 4\leq 9x-6$$

$4\leq 9x-6$ AND $-9x+6\leq 4$

$4+6\leq 9x$ $\qquad$ $-9x\leq 4-6$

$10\leq 9x$ $\qquad$ $-9x\leq -2$

$\frac{10}{9}\leq x$ $\qquad$ $x\geq\frac{2}{9}$

Thus, $\frac{2}{9}\leq x\leq\frac{10}{9}$

Example 12

Solve the equations below

a) $|x-3|+1 \leq 6$

b) $\left|\frac{2x+3}{x-1}\right| \geq 3$

SOLUTION:

a) $$|x-3|+1 \leq 6$$

$$|x-3| \leq 6-1$$

$$|x-3| \leq 5$$

$$-5 \leq x-3 \leq 5$$

$$-5 \leq x-3$$
$$-5+3 \leq x$$
$$-2 \leq x$$

AND

$$x-3 \leq 5$$
$$x \leq 5+3$$
$$x \leq 8$$

Thus, $-2 \leq x \leq 8$

$$b) \left|\frac{2x+3}{x-1}\right| \geq 3$$

$$\frac{2x+3}{x-1} \geq 3$$
$$2x+3 \geq 3(x-1)$$
$$2x+3 \geq 3x-3$$
$$2x-3x \geq -3-3$$
$$-x \geq -9$$
$$x \leq 9$$

OR

$$\frac{2x+3}{x-1} \leq -3$$
$$2x+3 \leq -3(x-1)$$
$$2x+3 \leq -3x+3$$
$$2x+3x \leq -3+3$$
$$5x \leq 0$$
$$x \leq 0$$

Thus, $x \leq 9 \ or \ x \leq 0$

Self Assessment

Exercise 2.1

Solve the following equations.

1. $4(x-4) = 3(x-2) - 1$

2. $\frac{1}{x+2} = \frac{1}{5}$

3. $\frac{(x+5)(x-1)}{4} = -2$

4. $\left|\frac{2+6x}{3x}\right| = 10$

Exercise 2.2

Solve the following inequalities.

1. $(x-3) \leq 2(x+2)$

2. $4(x-3) > 16 + 8x$

3. $x^2 - 3x + 2 > 0$

4. $|4x-3| < 7$

5. $\frac{3x-2}{x+1} \leq 2$

Answer

Exercise 2.1

Solve the following equations.

1. $x = 9$
2. $x = 3$
3. $x = -3 \quad x = -1$
4. $x = -\frac{1}{18} \quad x = \frac{1}{12}$

Exercise 2.2

Solve the following inequalities.

1. $x \leq 8$
2. $x < 7$
3. $x < 1 \; or \; x > 2$
4. $-\frac{7}{2} < x < 5$
5. $-1 \leq x \leq 4$

KEY TERM

Equation

Linear Function

Quadratic Function

Variable

Inequalities

Absolute value function

Rational

SURY

- Equations of function including Linear equation, quadratic equations, and absolute value equation
- inequalities of functions including Linear inequalities, quadratic inequalities, Rational inequalities and absolute value inequalities.
- Many simple inequalities can be solved by adding, subtracting, multiplying or dividing both sides until you are left with the variable on its own.

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2. Solving Inequalities – Explanation & Examples. (2021, March 17). The Story of Mathematics - A History of Mathematical Thought from Ancient Times to the Modern Day. https://www.storyofmathematics.com/inequalities-math

Arithmetic and Geometric progression

TOPIC 3

LEARNING OUTCOMES

By the end of topic, you should be able to:

1. Explain the terms of arithmetic progression and geometric progression.
2. Calculate the n-th term in arithmetic and geometric progression.
3. Calculate the sum of the first n-th terms in an arithmetic and geometric progression.

INTRODUCTION

A Sequence Or Progression is a succession of terms formed according to a certain rule or arranged in particular order.

Each number = a term of the sequence, separated by commas [series: plus or minus sign]

A Finite Progression	:	$a\,,\ T_2\,,T_3\,,\ldots\ldots\ldots\ldots\ldots\ldots\ldots,T_n$
A Finite Series	:	$a\ +\ T_2\ +\ T_3+\ldots\ldots\ldots\ldots\ldots\ldots+T_n$
An Infinite Progression	:	$a\,,T_2\,,T_3\,,\ldots\ldots\ldots\ldots$
An Infinite Series	:	$a\ +\ T_2\ +\ T_3\ +\ldots\ldots\ldots\ldots$

Some of the most common examples of sequences are Arithmetic equences Geometric Sequences. An Arithmetic Sequence is a sequence in which every term is created by adding or subtracting a definite number to the preceding number while for Geometric Sequences is a sequence in which every term is obtained by multiplying or dividing a definite number with the preceding number .

3.1 Arithmetic progression

In an arithmetic, there is a common difference between successive terms. Each term is obtained by add common difference to the preceding term.

An Arithmetic can be written as

a , a + d , a + 2d + ……………………., a + (n-1)d , ……………

Common difference, $\mathbf{d = T_n - T_{n-1}}$

The n-th term , $\mathbf{T_n = a + (n-1)\, d}$

Sum of the first n terms

$$\mathbf{S_n = \frac{n}{2}\ [\,2a + (n-1)\,d\,]}$$

OR

$$S_n = \frac{n}{2}\ [\,2a + (n-1)\,d\,]$$

$$= \frac{n}{2}\ [\,a + a + (n-1)d\,]$$

$$= \frac{n}{2}\ [\,a + T_n\,]$$

Sum of all term , $\mathbf{S_n = \frac{n}{2}}$ **[first term + last term]**

Example 1

Determine whether the following sequence of numbers is an arithmetic progression (A.P.) or not an arithmetic progression.

a. −5, −3, −1, 1, …
b. 3, 6, 15, 30, …

Solution:

a. −5, −3, −1, 1, …

d = T n − T n-1

$$T_2 - T_1 = T_3 - T_2$$
$$-3 - (-5) = (-1) - (-3)$$
$$2 = 2$$
$$Its\ Arithmetic\ progression$$

b. 3, 6, 12, 24, …

d = T n − T n-1

$$T_2 - T_1 = T_3 - T_2$$
$$6 - (3) = (15) - (6)$$
$$3 \neq 9$$
$$Its\ not\ Arithmetic\ progression$$

Example 2

Find the nth term for each of the following sequences.

a) 2, 4, 6, 8,...

b) 3, 5, 7, 9,...

Solution:

a)

$$a = 2$$
$$d = 4 - 2 = 2$$
$$Tn = a + (n - 1)d$$
$$Tn = 2 + (n - 1)2$$
$$Tn = 2 + 2n - 2$$
$$Tn = 2n$$

$b)$

$$a = 3$$
$$d = 5 - 3 = 2$$
$$Tn = a + (n - 1)d$$
$$Tn = 3 + (n - 1)2$$
$$Tn = 3 + 2n - 2$$
$$Tn = 2n + 1$$

Example 3

Given an Arithmetic Progression : -3, -5, -7, ….. Find

a) the first term and the common difference

b) the 10th term

c) the sum of the first 10 term

Solution:

a) First term, a = -3 Common difference, d = T3 – T2 = -7 – (-5) = -2

b) 10th term, T_{10} = a + 9d = -3 + 9(-2) = -3 + (-18) = - 21

c) Sum of the first 10 term, S10 = $\frac{10}{2}\,[-3\ +\ 9(-2)]$

$= \ 5\,[3\ +\ (-18)]$

$= -5(\,-21)\ =\ -\,105$

Example 4

The first term of an AP is 6 and the last term is 34. If the sum of all the terms is 800, calculate the number of terms and the common difference.

First term, a = 6 Last term, Tn = 34 sum of all term, Sn = 800

Solution:

$Sn\ = \frac{n}{2}\,[\,a\ +\ Tn\,]$

$\frac{n}{2}\,[\,6\ +\ 34\,]\ =\ 800$

$\frac{n}{2}\,[40]\ =\ 800$

$\frac{n}{2} = \frac{800}{40}$

$\frac{n}{2} =\ 20$

$n\ =\ 20\ \times\ 2\ =\ 40$

$T_{40} = a + 39d$

$34 = 6 + 39d$

$39d = 34 - 6$

$39d = 28$

$d = 28/39$

Example 5

The fourth term of an AP is 6 and the tenth term is 30. Find

a) The first term and the common difference

b) The sum of the first 17 terms of the series

Solution:

a) T4 = 6 T10 = 30

a + 3d = 6 ---------------(1) a + 9d = 30 ---------------------(2)

(2) – (1) : 9d – 3d = 30 – 6

6d = 24

$d = \frac{24}{6} = 4$

d = 4 (1) a + 3(4) = 6

a + 12 = 6

a = 6 – 12 = - 6

b) Sum of the first 17 term, S17 = $\frac{17}{2}$[2a + 16d)

$= \frac{17}{2}[\,2(-6) + 16(4)\,]$

$= \frac{17}{2}[-12 + 64]$

$= \frac{17}{2}[\,52\,] = 442$

3.2 Geometric progression

In a geometric, there is a common ratio between successive terms. Each term is obtained by multiply common ratio to the preceding term.

A geometric can be written as

a , ar , ar^2, ar^3 , ……………………

Common ratio, **r** **=** $\frac{T_n}{T_{n-1}}$ r ≠ 0 or 1

The n-th term Tn = a r^{n-1}

Sum of the first n terms

r > 1 : S_n = $\frac{a(r^n-1)}{r-1}$

r < 1 : S_n = $\frac{a(1-r^n)}{1-r}$

Sum to infinity exist only if -1 < r < 1 [|r| < 1 converges ; |r| > 1 @ r > 1 OR r < -1 Diverges]

$S_\infty = \frac{a}{1-r}$

Example 6

Find the nth term of the geometric sequence

5, 10, 20, 40,...

Solution:

5, 10, 20, 40,...

$a = 5\ r = \frac{10}{5} = 2$

$T_n = ar^{n-1}$

$T_n = (5)(2)^{n-1}$

Example 7

Given the Geometric Progression : $\frac{1}{81}, \frac{1}{27}, \frac{1}{9}, \ldots$..Find

a) the first term and the common ratio

b) the 8th term

c) the sum of the first 6 terms

Solution:

a) First term, a = $\frac{1}{81}$ Common ratio, r = $\frac{T3}{T2} = \frac{\frac{1}{9}}{\frac{1}{27}} = \frac{1}{9} \times \frac{27}{1} = 3$

b) Eight term. $T_8 = ar^7 = (\frac{1}{81})(3)7 = (\frac{1}{81})(2187) = 27$

c) Sum of the first 6 term. S6 = $\frac{a(r^6-1)}{r-1} = \frac{\frac{1}{81}[3^6-1]}{3-1} = \frac{\frac{1}{81}[729-1]}{2}$

$$= \frac{\frac{1}{81}[728]}{2} = \frac{364}{81}$$

Example 8

Given the Geometric Progression : 3, 9, 27 81, … , 2 187 Find

a) the first term and the common ratio

b) the 5th term

c) Find the number of terms, Tn for this progression

c) the sum of the first 6 terms

Solution:

a) the first term and the common ratio

$a = 3, r = \frac{T_2}{T_1} = \frac{9}{3} = 3$

b) Find the 5th terms.

n=5

$$T_n = ar^{n-1}$$
$$T_5 = (3)(3)^{5-1}$$
$$T_5 = 3(3^4)$$
$$T_5 = 3(81)$$
$$= 243$$

c) Find the number of terms, Tn for this progression

$$T_n = ar^{n-1}$$

$$2\,187 = (3)(3)^{n-1}$$
$$\frac{2187}{3} = 3^{n-1}$$

$$729 = 3^{n-1}$$
$$3^6 = 3^{n-1}$$
$$6 = n - 1$$
$$6 + 1 = n$$
$$n = 7$$

d) Sum of the first 6 term, $S_6 = \frac{a[1-r^6]}{1-r} = \frac{3\,[1-[3]^6)}{1-3} = \frac{3(1-729)}{-2}$

$$= \frac{3(-728)}{-2} = \frac{-2184}{-2} = 1{,}092$$

Example 9

Given the Geometric Progression : $1, \frac{3}{2}, \frac{9}{4}, \frac{27}{8}, \ \ldots, \ \frac{19\,683}{512}$. Find

a) the first term and the common ratio

b) the 8th term

c) Find the number of terms, Tn for this progression

c) the sum of the progression

Solution:

a) the first term and the common ratio

a=1

r=$\frac{\left(\frac{3}{2}\right)}{1}=\frac{3}{2}$

b) Find the 8th terms.

$$T_n = ar^{n-1}$$

$$T_8 = (1)\left(\frac{3}{2}\right)^{8-1}$$

$$T_8 = 1\left(\left(\frac{3}{2}\right)^7\right)$$

$$T_8 = 1\left(\frac{2187}{128}\right) = \frac{2187}{128}$$

c) Find the number of terms, Tn for this progression

$$T_n = ar^{n-1}$$

$$\frac{19\,683}{512} = (1)\left(\frac{3}{2}\right)^{n-1}$$

$$\frac{\left(\frac{19\,683}{512}\right)}{1} = \left(\frac{3}{2}\right)^{n-1}$$

$$\frac{19\,683}{512} = \left(\frac{3}{2}\right)^{n-1}$$

$$\left(\frac{3}{2}\right)^9 = \frac{3^{n-1}}{2}$$

$$9 = n - 1$$

$$9 + 1 = n$$

$$n = 10$$

d) the sum of the progression

$$S_{10} = \frac{1(1-(\frac{3}{2})^{10})}{1-(\frac{3}{2})} = \frac{3(-56.67)}{(-\frac{1}{2})} = 340.02$$

Example 10

The first term and the sum to infinity of geometric progression are 24 and 36 respectively. Find the common ratio of the progression.

Solution:

$a = 24 \quad , S_\infty = 36$

$\frac{24}{1-r} = 36 \quad \rightarrow \quad 24 = 36 - 36r$

$36r = 12$

$r = \frac{1}{3}$

Self Assessment

Exercise 3.1

1. Given the arithmetic progression 4,7,10.. 61.
   a. State the value of a and d
   b. Find the sum of the terms of the series

2. The first three terms of an AP are 83, 79, and 75. Find the 22th term.

3. The fifth term of an arithmetic progression is 22 and the ninth term is 50, find :
   a) The first term and the common difference
   b) The sum of the first ten terms

Exercise 3.2

1. Given geometric progression is 2,-6,18,... . Find the sum of the first 8th terms of each of the following progression.

2. Given the Geometric Progression : $1, \frac{1}{2}, \frac{1}{4} \dots, \frac{1}{512}$. Find

   a)the first term and the common ratio
   b) the 4th term
   c) Find the number of terms, Tn for this progression

3. The first term and the sum to infinity of geometric progression are $\frac{2}{5}$ and 45 respectively. Find the first term of the progression.
4. Find the number of terms in for this geometric progression:

$$32, 48, 72, \dots, \frac{6561}{8}$$

Answer

EXERCISE 3.1

1. $a. a = 4 \quad d = 3$
   $b.\ S_{20} = 650$

2. $T_{22} = -1$

3. $a = 50 \quad d = -7$
   $s_{10} = 815$

EXERCISE 3.2

1. $-3\,280$
2. a) $a = 1, r = \frac{1}{2}$
   b) $T_8 = \frac{1}{16}$
   c) n=10
3. 27
4. $9th\ Term$

KEY TERM

Arithmetic Progression

Geometric Progression

Terms

First terms

Common Difference

Common Ratio

SUMMARY

- Arithmetic Sequences are those in which each term is achieved by multiplying or dividing a definite number with the preceding number, whereas Geometric Sequences are those in which each phrase is produced by multiplying or dividing a defined number with the preceding number.
- In Arithmetic progression, The n-th term , $T_n = a + (n-1)\,d$ while Sum of the first n terms is $S_n = \frac{n}{2}\,[\,2a + (n-1)\,d\,]$.
- In Geometric progression, The n-th term , $T_n = a\,r^{n-1}$ while Sum of the first n terms is $r > 1 : S_n = \frac{a(r^n-1)}{r-1}$ and $r < 1 : S_n = \frac{a(1-r^n)}{1-r}$.
-

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INDICES AND LOGARITHMS

TOPIC 4

LEARNING OUTCOMES

By the end of topic, you should be able to:

1. Slimplify algebraic expression using laws of indices and logarithms.
2. Solve the equations involving indices and logarithms.

INTRODUCTION

An index number is a number which is raised to a power. The power, also known as the index, tells you how many times you have to multiply the number by itself. It is written as a small number to the right and above the base number.

For example :

$$a^n$$

where a is base and n is exponene or power.

While for logarithms, a positive number N to a given base a (a> 0,a≠ 1) is the power to which the base a must be raised in order to give the number.

$$a^x = N \leftrightarrow log_a N = x$$

4.1 INDICES

B. INDICES POSITIVE & NEGATIVE

$\boldsymbol{a^n} = n^{th}\, power\, of\, a \;\; which\, is\, [\, a^2 = square/a^3 = cubic]$

$\boldsymbol{a^n = a \times a \times a \ldots a}$ [multiply of a by n times @ a multiplied n times @ the product of a with itself n times]]

$$a^{-n} = reciprocal\, of\, positive\, indices$$

$$a^{-n} = \frac{1}{a^n}$$

B. INDICES ZERO

$$: a\, 0 = 1$$

C. INDICES FRACTIONAL

$\sqrt[n]{a} = a^{\frac{1}{n}}$: $principal\, n^{th}\, root\, of\, a\, [\sqrt{a} = \;\; square\, root$

: $\sqrt[3]{a}\, cubic\, root\, @\, a\, is\, the\, radicand\, and\, \sqrt{\;}\;\; is$

$a\, radical]$

$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{(a^m)}: n^{th}\, root\, for\, a^m\, OR\, m^{th}\, power\, of\, \sqrt[n]{a}$

LAWS OF INDICES

1. $(a^m)^n = a^{mn}$

2. $a^m \times a^n = a^{m+n}$

3. $a^m \div a^n = a^{m-n}$

4. $(ab)^m = (a^m b^m)$

5. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$\

NO LAWS OF INDICES FOR DIFFERENT BASE AND DIFFERENT INDICES

1) $a^m \times b^n \neq (a \times b)^{m+n}$
2) $a^m \div b^n \neq (a \div b)^{m+n}$
3) $a^n \pm b^n \neq (a \pm b)^n$

Example 1

Simplify each of the following expression

a) $(m^2 n^4)^2$ b) $a^6 a^{-5} a^4$

c) $(x^2\, y^2) \times (x^3 y^2)$(d) $(x^4\, z^6) \div (xz)^3$

Solution:

a. $(m^2 n^4)^2 = m^{2\times2} n^{4\times2} = m^4 n^8$
b. $a^6 a^{-5} a^4 = a^{6+(-5)+4} = a\,5$
c. $(x^2\, y^2) \times (x^3 y^2) = x^{2+3}\, y^{2+2} = x^5\, y^4$
d. $(x^4\, z^6) \div (xz)^3 = (x^4\, z^6) \div (x^3 z^3) = x^{4-3}\, z^{6-3} = x^1\, y^3$

Example 2

Simplify the following expressions.

a) $(2x^{-1}y)^2 \times (x^1y^4)$

b) $\frac{(xy)^{-2}}{x^{-3}y^2}$

Solution:

a) $(2x^{-1}y)^2 \times (x^1y^4) = \left(2^2x^{-2}y^2\right) \times (x^1y^4) = 2^2x^{-2+1}\, y^{2+4} = 4x^{-1}y^6 = \frac{4y^6}{x^1}$

b) $\frac{(xy)^{-2}}{x^{-3}y^2} = \frac{x^{-2}y^{-2}}{x^{-3}y^2} = x^{-2-(-3)}\, y^{-2-(-2)} = x^1y^0 = x^1$

Example 3

Simplify the following expressions.

$a)\ \left(-\frac{1}{2}\right)^3$

$b)\ \frac{(pq)^{-1}}{p^{-1}+q^{-1}}$

a) $\left(-\frac{1}{2}\right)^3 = (-1)^3\left(\frac{1}{2}\right)^3 = -\frac{1}{8}$

$b)\ \frac{(pq)^{-1}}{p^{-1}+q^{-1}} = \frac{\frac{1}{pq}}{\frac{1}{p}+\frac{1}{q}} = \frac{1}{pq} \times \frac{pq}{p+q} = \frac{1}{p+q}$

4.2 LOGARITHMS

- A log to base 10 = common logarithm written as $\log_{10} a$ or lg
- A log to base e = natural logarithm [OR Naperian logarithm] written as **ln**
- The logarithm of a NEGATIVE number or ZERO is undefined / does not exist
- Conversation for exponent to log and vice verse

Exponent	:	$y = a^x$
Log	:	$\log_a y = x$

Laws of Logarithms

1) $1 = a^0$

$\log_a 1 = 0$ log of 1 to any base is 0

2) $a = a^1$

$\log_a a = 1$ log of any number to the same value of base is 1

3) $\log_a (xy) = \log_a x + \log_a y$

4) $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$

5) $\log_a (x^m) = m \log_a x$

IT IS WRONG WHEN

1) $\log_a p \pm \log_a q \neq \log_a (p \pm q)$

2) $\log_a \left(\frac{p}{q}\right) \neq \frac{\log_a p}{\log_a q}$

3) $\log_a (pq) \neq \log_a p \times \log_a q$

4) $\log_a (x^m) \neq (\log_a x)^m$

5) $\frac{\log_a p}{\log_a q} \neq \log_a p - \log_a q$

Example 4

State the following expressions in single logarithmic expression

(1) $\log_2 5 + \log_2 8 - \log_2 20$

(2) $\log_3 7 + \log_3 18 - \log_3 14$

(3) $\log_5 125 + \log_5 3 - \log_5 15$

(4) $2 \log_3 5 + \log_3 4 - \log_3 10$

Solution:

1) $\log_2 5 + \log_2 8 - \log_2 20 = \log_2 \left(\frac{5\times8}{20}\right) = \log_2 2 = 1$

2) $\log_3 7 + \log_3 18 - \log_3 14 = \log_3 \left(\frac{7\times18}{14}\right) = \log_3 9 = \log_3 3^2$

$= 2 \log_3 3 = 2(1) = 2$

3) $\log_5 125 + \log_5 3 - \log_5 15 = \log_5 \left(\frac{125\times3}{15}\right) = \log_5 25 = \log_5 5^2$

$= 2 \log_5 5 = 2(1) = 2$

4) $2 \log_3 5 + \log_3 4 - \log_3 10 = \log_3 5^2 + \log_3 4 - \log_3 10$

$= \log^3 \left(\frac{25\times4}{10}\right) = \log_3 10$

4.3 SOLVING EQUATION OF INDICES & LOGARITHM

EQUATION OF INDICES

KEY 1: COMPARISON METHOD IF INDICE IS VARIABLE

Express both sides of equation in terms as SAME BASE

If $a^x = k$ [EXPRESS TO SAME BASE COMPARE INDICES]

$a^x = a^y$

$x = y$

KEY 2: COMPARISON METHOD IF BASE IS VARIABLE

Express both sides of equation in terms as SAME INDICES

$x^a = k$ [EXPRESS TO SAME INDICES COMPARE BASE]

$x^a = y^a$

$x = y$

OR $x^a = k$

$x = \sqrt[a]{k}$

OR $\sqrt[n]{x} = k$

$x = k^n$

Example 5

Simplify the following expressions.

$$2^{2n} \times 8^{3n} = 4^{5n+1}$$

Solution:

$$2^{2n} \times 2^{3(3n)} = 2^{2(5n+1)}$$
$$2^{2n+9n} = 2^{10n+2}$$
$$2n + 9n = 10n + 2$$
$$2n + 9n - 10n = 2$$
$$n = 2$$

Example 6

Simplify the following expressions

$$243^{x-2} = \frac{1}{27^x}$$

Solution:

$243^{x-2} = \frac{1}{27^x}$

$3^{5(x-2)} = \frac{1}{3^{3x}}$

$3^{5x-10} = 3^{-3x}$

$5x - 10 = -3x$

$5x + 3x = 10$

$8x = 10$

$x = \frac{5}{8}$

$x = \frac{5}{4}$

Example 7

Solve

1) $2^x = 8$ 2) $3^x = 81$ 3) $2x^4 = 32$

4) $2^{y+3} = 2^{5y}$ 5) $2^{2y-13} = 2^{y+7}$ 6) $2^{2x-14} = 2^{x+6}$

Solution:

1)

$2^x = 8$

$2^x = 2^3$

$x = 3$

2)

$3^x = 81$

$3^x = 3^4$

$x = 4$

3)

$2x^4 = 32$

$x^4 = \frac{32}{2}$

$x^4 = 16$

$x^4 = 2^4$

$x = \sqrt[4]{16}$

$x = 2$

4)

$2^{y+3} = 2^{5y}$

$y + 3 = 5y$

$y - 5y = -3$

$-4y = -3$

5)

$2^{2y-13} = 2^{y+7}$

$2y - 13 = y + 7$

$2y - y = 7 + 13$

$y = 20$

6)

$2^{2x-14} = 2^{x+6}$

$2x - 14 = x + 6$

$2x - x = 6 + 14$

$x = 20$

Example 8

Find the value of x for the following equations

1) $\log_2 x = 4$ 2) $\log_x 64 = 3$ 3) $\log_4(8x) = 2$

4) $\log_4 x - \log_4 7 = \frac{3}{2}$ 5) $log\, 3\,(x+8) - \log 3\, x = 2$

6) $2\log_4 3 + \log_4 2x = \log_4(3x+1)$

Solution:

1) $\log_2 x = 4$

$x = 4^2$

$x = 16$

2) $\log_x 64 = 3$

$x^3 = 64$

$x = \sqrt[3]{64} = 4$

3) $\log_4(8x) = 2$

$8x = 4^2$

$8x = 16$

$x = \frac{16}{8} = 2$

4) $\log_4\left(\frac{x}{7}\right) = \frac{3}{2}$

$\frac{x}{7} = 4^{\frac{3}{2}}$

$\frac{x}{7} = 8$

$x = 56$

5) $log\, 3\,(x+8) - \log 3\, x = 2$

$\log_3\left(\frac{x+8}{x}\right) = 2$

$\frac{x+8}{x} = 3^2$

$x + 8 = 9x$

$x - 9x = -8$

$-8x = -8$

$x = -\frac{8}{-8}$

$x = 1$

6) $2\log_4 3 + \log_4 2x = \log_4(3x+1)$

$\log_4(3^2 \times 2x) = \log_4(3x+1)$

$9(2x) = (3x+1)$

$18x - 3x = 1$

$15x = 1$

$x = \frac{1}{15}$

Self Assessment

Exercise 4.1

1. Simplify

a) $\frac{(x^4 z^6)}{(xz)^3}$ b) $\frac{x^2 y^3}{x^{-3} y^4}$ c) $\frac{a^{3m+2} \times a^{4-m}}{a^{3+4m}}$ d) $3^{2x-2} = 9^{2x-1}$

Exercise 4.2

1. Solve
   a. $\log_9 81 = x$
   b. $\log_3 \left(\frac{1}{81}\right) = x$

2. $2\log_4(5) + 2\log_4(4) - 2\log_4(10) = x$

3. $\log_2 6 - \log_2 3 + \log_2 2x = \log_2(3x + 5)$

4. $\log_2 \left(\frac{2}{x}\right) - \log_2 x = 3$

Answer

Exercise 4.1

1. Simplify

a) x^1z^3 $b)\frac{(x^5)}{y^1}$ c) a^{-2m+3} d) $x = 0$

Exercise 4.2

1. Solve
   a. $x = 2$
   b. $x = -4$
2. $x = 1$
3. $x = 5$
4. $x = \pm\frac{1}{2}$

KEY TERM

Indices	**Logarithms**
Exponential	**base**

SUMMARY

Law of indices including:

1. $(a^m)^n = a^{mn}$ 2. $a^m \times a^n = a^{m+n}$ 3. $a^m \div a^n = a^{m-n}$

4. $(ab)^m = (a^m b^m)$ 5. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Law of logarithms including:

1) $\log_a (xy) = \log_a x + \log_a y$ 2) $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$

3) $\log_a (x^m) = m \log_a x$ 4) $\log_a a = 1$

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TOPIC 5

FUNCTION

LEARNING OUTCOMES

By the end of topic, you should be able to:

1. Explain domain and range, operations on functions, odd and even functions.
2. Identify composite functions and inverse functions

INTRODUCTION

Relation is used to show the connection between the elements of 2 set (Inputs and Outputs). A function is a rule that receives an input and produces an output (a rule that assigns to each input exactly one output). Only one output is produced for any given input.

A function is a relation that express how one quantity (output) depends on another quantity (input).A function is usually specified by a formula that shows what must be done to the input to produce the output.

The input is referred to as the independent variable. The output is referred to as dependent variable because its value depends upon and uniquely determined by the value of input.

Written as f (input) = output $f(x) = y$

[*f(x) DOES NOT MEAN f times x BUT the output that correspond to the input x.]

To obtain output when input is k , replace the each input in the function by k and evaluate. The domain of the function is the set of values of independent variable, x (inputs/object) and the range/co-domain of the function is the set of values of dependent variable,y (outputs/image).

5.1 Domain and Range

For Function y=f(x)

Domain, D_f is the set of all values of x for which function f is defined.

Range, R_f is the set of all possible values of f(x) for each value of x in the domain.

Example 1

Find the domain and range for function:

a) $f(x) = 4x - 1$

b) $f(x) = x^2 + 4$

c) $f(x) = \sqrt{x-4}$

d) $f(x) = \frac{2}{x}$

Solution:

a) $\boldsymbol{f(x) = 4x - 1}$

$\boldsymbol{Df = (x\text{: } x \in R)}$

$\boldsymbol{Rf = (y\text{: } y \in R)}$

b) $\boldsymbol{f(x) = x^2 + 4}$

$\boldsymbol{D}\text{f} = (x\text{: } x \in R)$

To find Range for quadratic function:

Find maximum or minimum point using quadratic function $(ax^2 + bx + c = 0)$

1st Step: *Find value x first using* $\frac{-b}{2a}$

2nd Step: Then, substitute value of x into the function given.

3rd Step: Lastly, you will get the value of y.

Thus, $f(x) = x^2 + 4$, $where\ a = 1\ and\ b = 0$

$$x = \frac{-b}{2a} = -\frac{0}{2(1)} = 0$$

When x = 0 thus,

$f(0) = 0^2 + 4 = 4$ $\quad\quad$ $\boldsymbol{Rf} = [\boldsymbol{y}: \boldsymbol{y} \in \boldsymbol{R}, \boldsymbol{y} \geq \mathbf{4}]$

c) $\boldsymbol{f(x)} = \sqrt{\boldsymbol{x-4}}$

$\boldsymbol{Df} = x - 4 \geq 0$

$x \geq 4$

$\boldsymbol{Df} = (\boldsymbol{x}: \boldsymbol{x} \in \boldsymbol{R}, \boldsymbol{x} \geq \mathbf{4})$

$\boldsymbol{Rf}$ = since $\sqrt{x-4} \geq 0$

$f(x) \geq 0$

$\boldsymbol{Rf} = [\boldsymbol{y}: \boldsymbol{y} \in \boldsymbol{R}, \boldsymbol{y} \geq \mathbf{0}]$

d) $\boldsymbol{f(x)} = \frac{2}{x}$

$\boldsymbol{Df} = \boldsymbol{x} \neq \mathbf{0}$

$x \neq 0$

$\boldsymbol{Df} = (\boldsymbol{x}: \boldsymbol{x} \in \boldsymbol{R}, \boldsymbol{x} \neq \mathbf{0})$

Range = Do let y (inverse function)

Let y $= \frac{2}{x}$

$y(x) = 2$

$x = \frac{2}{y}$

$f^{-1}(x) = \frac{2}{y}$

$\boldsymbol{Rf} = [\boldsymbol{y}:\ \boldsymbol{y} \in \boldsymbol{R},\ \boldsymbol{y} \neq \boldsymbol{0}]$

5.2 Operation on Function

OPERATION	TERMINOLOGY	FUNCTION VALUE
Sum	*f(x) + g (x)*	*(f + g)(x) = f (x) + g (x)*
Difference	*f(x) - g (x)*	*(f – g)(x) = f(x) – g(x)*
Product	*[f(x)] [g (x)]*	*(fg)(x) = f(x)g(x)*
Quotient	$\frac{f(x)}{g(x)}$	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

Example 2

Given the functions f (x) = $x^2 + 2$ and g (x) = 3x + 1. Find the functions

a) (f + g)(x) b) (f – g)(x) c) (fg)(x) d) (f/g)(x)

Solution:

a) $(f + g)(x) = f(x) + g(x) = (x^2 + 2) + (3x + 1) = x^2 + 2 + 3x + 1$

$= x^2 + 3x + 3$

b) $(f - g)(x) = f(x) - g(x) = (x^2 + 2) - (3x + 1) = x^2 + 2 - 3x - 1$

$= x^2 - 3x + 1$

c) $(fg)(x) = f(x)\ x\ g(x) = (x^2 + 2) \times (3x + 1) = 3x^3 + x^2 + 6x + 2$

d) $(f/g)(x) = \frac{f(x)}{g(x)} = \frac{x^2+2}{3x+1}$

Example 3

The function f is given by $f : x \to 4x - 5$. Find

a) $f(3)$

b) the value of x when $f(x) = 7$

c) the value of x such that $f(x) = x$

Solution:

a) $f(x) = 4x - 5$

$f(3) = 4(3) - 5 = 12 - 5 = 7$

b) $f(x) = 4x - 5 = 7$

$4x = 7 + 5$

$4x = 12$

$x = 3$

c) $f(x) = 4x - 5 = x$

$4x - x = 5$

$3x = 5$

$x = \frac{5}{3}$

Example 4

The function g is given by $g : x \to ax + b$. If $g(-2) = 5$ and $g(3) = 10$, find the

Solution:

$g(x) = ax + b$

$g(-2) = a(-2) + b = 5$

$-2a + b = 5$ ----------------(1)

$g(3) = a(3) + b = 10$

$3a + b = 10$ -----------------------(2)

(2) – (1) : $(3a + b) - (-2a + b) = 10 - 5$

$3a + 2a = 5$

$5a = 5$

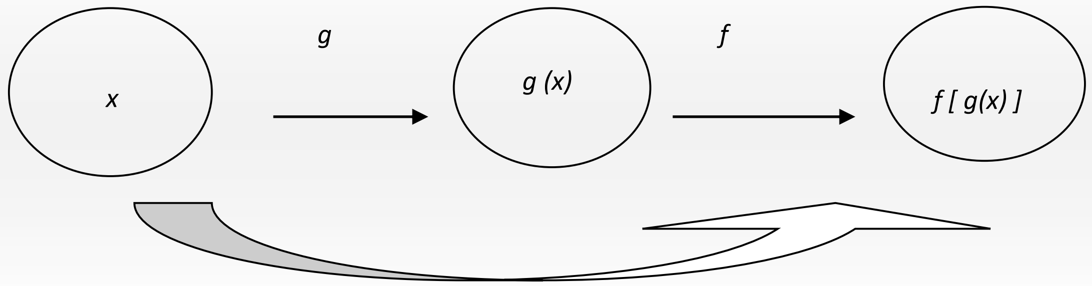
$a = 1$

$-2(1) + b = 5$

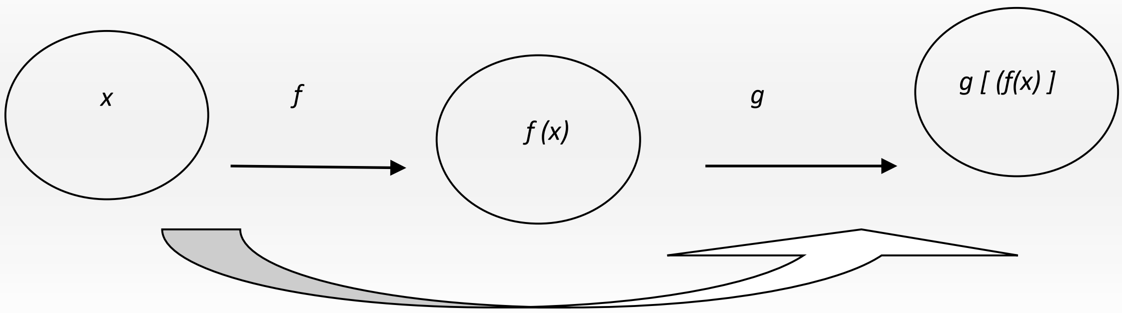
$-2 + b = 5$

$b = 5 + 2 = 7$

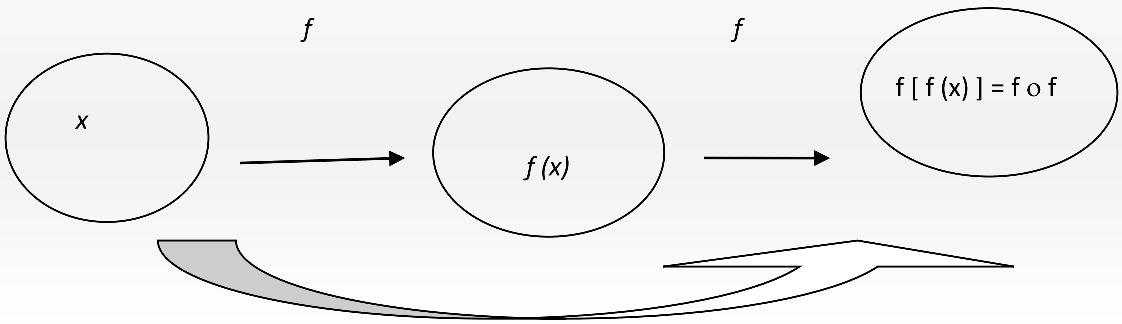
5.3 Composite Function



$f \circ g = f\,[\,g(x)\,]$



$g \circ f = g\,[\,f(x)\,]$



$f \circ f = f\,[\,f(x)\,] = f^{2}(x)$

Example 5

Given the functions f : x ⟶ 2x + 3 and g : x ⟶ x^2. Find

a) (f ○ g)(x)   b) (g ○ f)(x)

c) the value of (f ○ g)(1)   d) the value of (g ○ f)(2)

e) the value of x if (f ○ g)(x) = 5x

f) the value of x if (g ○ f)(x) = 9

Solution:

a) $(f \circ g)(x) = f[g(x)] = f(x^2) = 2(x^2) + 3 = 2x^2 + 3$

b) $(g \circ f)(x) = g[f(x)] = g(2x + 3) = (2x + 3)^2$

c) $(f \circ g)(1) = 2(1)^2 + 3 = 2(1) + 3 = 2 + 3 = 5$

d) $(g \circ f)(2) = [2(2) + 3]^2 = [4 + 3]^2 = 7^2 = 49$

e) $(f \circ g)(x) = 2x^2 + 3 = 5x$

$$2x^2 - 5x + 3 = 0$$

$$(2x - 3)(x - 1) = 0$$

$$2x - 3 = 0 \quad or \quad x - 1 = 0$$

$$x = \frac{3}{2} \qquad x = 1$$

f) $(g \circ f)(x) = (2x + 3)\ 2 = 9$

$$2x + 3 = \sqrt{9}$$

$$2x + 3 = 3 \quad or \quad 2x + 3 = -3$$

$$2x = 3 - 3 \qquad 2x = -3 - 3$$

$$2x = 0 \qquad 2x = -6$$

$$x = 0 \qquad x = -3$$

Example 6

Given $f(x) = 3x - 1$ and $g(x) = \frac{x}{2}$. Determine the composite functions

i) fg(x) ii) gf(x) iii) gf(2) iv) fg(2)

Solution:

i) $(f \circ g)(x)$

$= f\left(\frac{x}{2}\right)$

$= 3\left(\frac{x}{2}\right) - 1$

$= \frac{3}{2}x - 1$

ii) $(g \circ f)(x)$

$= g(3x - 1)$

$\frac{3x - 1}{2}$

iii) $(g \circ f)(2)$

$f(2) = 3(2) - 1 = 4$

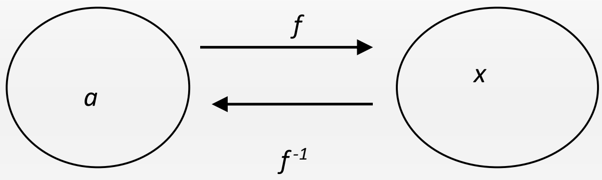
$g(f(2)) = \left(\frac{4}{2}\right) = 2$

iv) $(f \circ g)(2)$

$g(2) = \left(\frac{2}{2}\right) = 1$

$f(g(2)) = 3(1) - 1 = 2$

5.4 Inverse Function



f (input) = output $\qquad$ $f(a) = x$

f^{-1} *(output) = input* $\qquad$ $f^{-1}(x) = a$

- The inverse function f exists if and only if f is a one to one function
- The graph of $y = f^{-1}(x)$ is the reflection of the graph $y = f(x)$ in the line $y = x$
- Domain of f^{-1} is the range of f. Range of f^{-1} is the domain of f.

Example 7

Find the inverse of each of the following function

a) $f(x) = 5x - 2$ $\qquad$ b) $f(x) = \sqrt{x+3}$ $\qquad$ c) $f(x) = x3 + 4$

d) $f(x) = x^2 + 5$

Solution:

a)

$$f(a) = 5a - 2 = x$$
$$5a = x + 2$$
$$a = \frac{x+2}{5}$$
$$f^{-1}(x) = \frac{x+2}{5}$$

b)

$$f(a) = \sqrt{a+3} = x$$
$$a + 3 = x^2$$
$$a = x^2 - 3$$
$$f^{-1}(x) = x^2 - 3$$

c)

$$f(a) = a^3 + 4 = x$$
$$a^3 = x - 4$$
$$a = \sqrt[3]{x-4}$$
$$f^{-1}(x) = \sqrt[3]{x-4}$$

d)

$$f(a) = a^2 + 5 = x$$
$$a^2 = x - 5$$
$$a = \sqrt{x-5}$$
$$f^{-1}(x) = \sqrt{x-5}$$

Example 8

The function g is given by g : x → ax + b. If g^{-1}(-2) = 4 and g^{-1} (3) = 9, find the values of a and b.

Solution:

$g\,(x) = ax + b$

$g^{-1}\,(-2) = 4$ $\qquad$ $g^{-1}\,(3) = 9$

$g\,(4) = -2$ $\qquad$ $g\,(9) = 3$

$g(4) = a(4) + b = -2$ $\qquad$ $g\,(9) = a(9) + b = 3$

$4a + b = -2 ----(1)$ $\qquad$ $9a + b = 3 ----(2)$

$(2) - (1):$ $\quad (9a + b) - (4a + b) = 3 - (-2)$

$9a - 4a = 3 + 2$

$5a = 5$

$a = 1$

$a = 1$ $\quad (1) \quad 4(1) + b = -2$

$b = -2 - 4 = -6$

Self Assessment

Exercise 5.1

Given the functions f : x → $x + 4$ and g : x → $3x^2 + 1$. Find

a) $(f \circ g)(x)$ b) $(g \circ f)(x)$

c) the value of $(f \circ g)(1)$ d) the value of $(g \circ f)(2)$

e) the value of x if $(f \circ g)(x) = 9x$

f) Find the Domain and Range of f(x)

g) Find the inverse function of g(x)

Exercise 5.2

Given the functions f : x → $\frac{3x}{x-1}$ and g : x → $2x - 1$. Find

a) $(f \circ g)(x)$ b) $(g \circ f)(x)$

c) the value of $(f \circ g)(1)$ d) the value of $(g \circ f)(2)$

e) the value of x if $(g \circ f)(x) = 2$

f) Find the Domain and Range of g(x)

g) Find the inverse function of f(x)

Answer

Exercise 5.1

a) (f ∘ g)(x)=$3x^2 + 5$

b) (g ∘ f)(x)=$3x^2 + 24x + 49$

c) the value of (f ∘ g)(1) = 8

d) the value of (g ∘ f)(6) = 301

e) the value of x if (f ∘ g)(x)

$x_1 = 2.264$

$x_2 = 0.736$

f) Domain f(x) $(x\colon x \in R)$

Range f(x) $(y\colon y \in R)$

g) $g^{-1}(x) = \sqrt{\frac{y-1}{3}}$

Exercise 5.2

Given the functions f : x ⟶ $\frac{3x}{x-1}$ and g : x ⟶ $2x - 1$. Find

a) (f ∘ g)(x) $= \frac{6x-3}{2x-2}$

b) (g ∘ f)(x) $= \frac{5x+1}{x-1}$

c) the value of (f ∘ g)(2)= $\frac{9}{2}$

d) the value of (g ∘ f)(2)= 11

e) the value of x if (g ∘ f)(x) = 2

$x = -1$

f) Find the Domain and Range of g(x)

Domain g(x) $(x\colon x \in R)$

Range g(x) $(y\colon y \in R)$

g) $f^{-1}(x) = \frac{1}{x-3}$

KEY TERM

Arithmetic Progression

Geometric Progression

Terms

First terms

Common Difference

Common Ratio

SUMMARY

- Functions that involve are Linear function, Quadratic functions, Surds function and Reciprocal function.
- Domain is the values of x and range is the value of yand operations on functions including addition, subtraction, multiplication and division
- Function have composite functions and inverse functions

REFERENCEES

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2. Algebra - Inverse Functions. (2021). Paul's Online Note. *https://tutorial.math.lamar.edu/classes/alg/inversefunctions.aspx*

3. Swokowski, E., & Cole, J. (2018). Precalculus: Functions and Graphs (13th ed.) [E-book]. Cengage Learning.

GRAPH OF FUNCTION

TOPIC 6

LEARNING OUTCOMES

By the end of topic, you should be able to:

1. Identify the graph of linear, quadratic, cubic and reciprocal functions.
2. Solving the equation of graph of linear, quadratic, cubic and reciprocal functions.

INTRODUCTION

A coordinate graph can be used to describe a function where both the input (independent variable) and output (dependent variable) are real numbers. The input is plotted horizontally on the x-axis, and the output is plotted vertically on the y-axis.

First of all you need to know the steps to draw a graph. The steps involve:

1. Construct a tble of values for a chosen range of x values
2. Draw the x axis and the y axis and use a suitable scale for each axis starting from the origin
3. plot the x and y values as coordinates pairs on the cartesian pane
4. Join the points to form a straight line (using a ruler) or a smooth curve (without using a ruler).
5. Label the graph

Diagram 6.1 showed the diagram of x-axis and y axis:

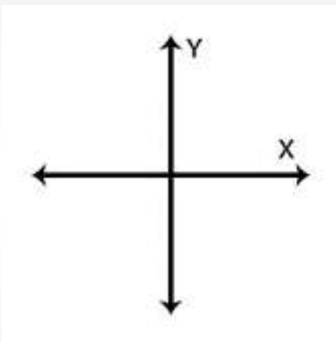


Diagram 6.1: x axis and y axis

Graph of function that involve is:

- Graph of linear function

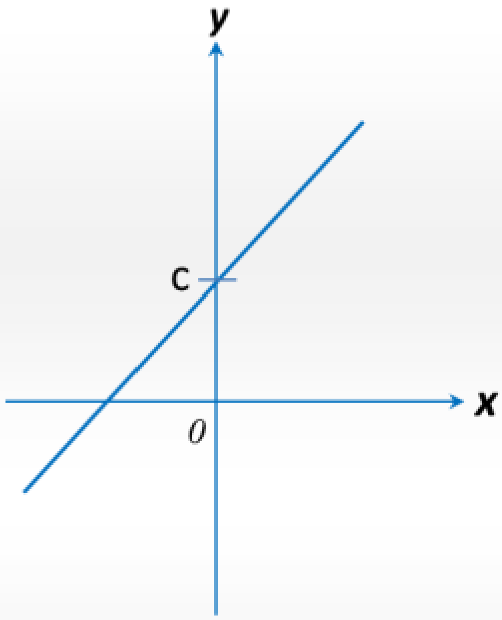
- Graph of quadratic function
- Graph of cubic function
- Graph of reciprocal function

6.1 Graph function

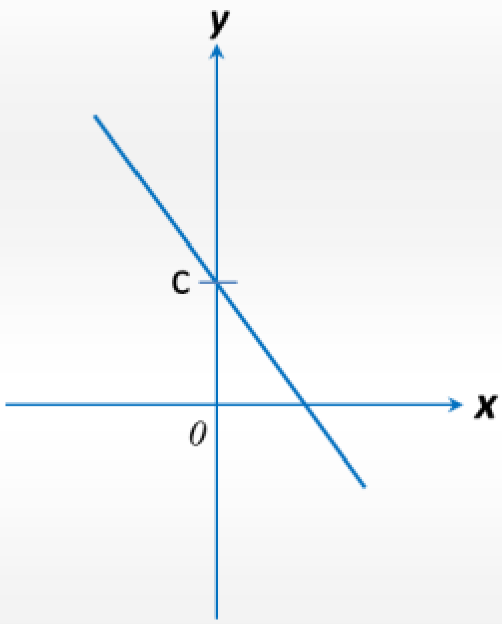
A. Linear Function

The graph of a linear function is a straight line. Often written as y=ax+c. One independent variable and one dependent variable make up a linear function. The dependent variable is y, while x is the independent variable. The constant term, often known as the y intercept, is denoted by the letter a. Highest power of the variable *x* is **1**.

i) y= ax+c , a>0 and c is y intercept



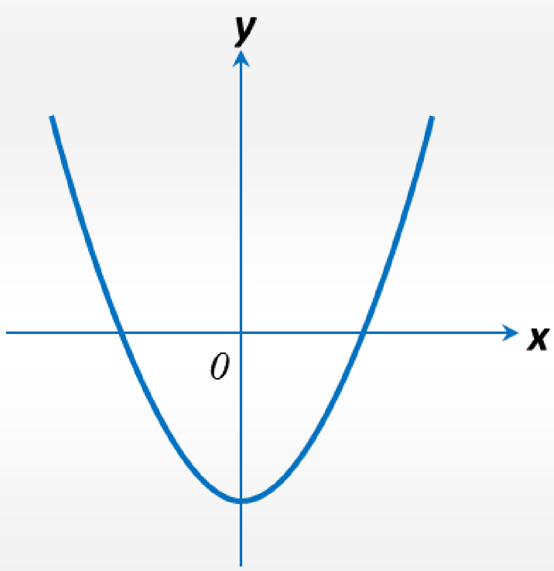
ii) y= -ax+c , a<0 and c is y intercept



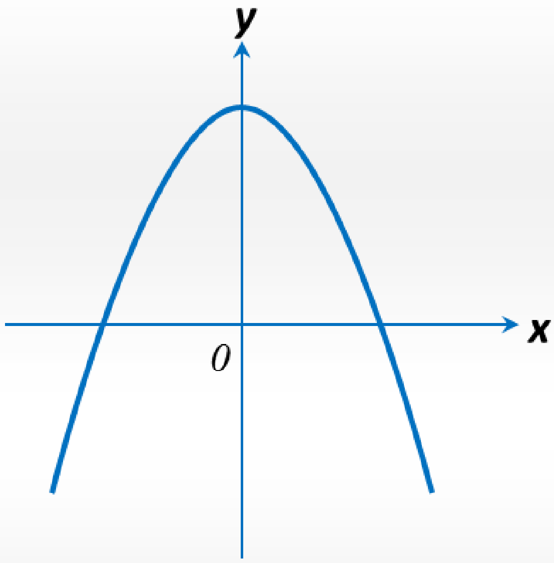
B. Quadratic Function

A parabola is a U-shaped curve that represents a quadratic function. Usually represent in y= $ax^2 + bx + c$ *where b and c is constant*. The sign of the quadratic function's coefficient a determines whether the graph expands up or down. If a<0 is 0, the graph frowns (closes down), and if a>0, the graph smiles (opens up). Highest power of the variable x is 2.

i) y= $ax^2 + bx + c, a > 0$ *and c is y intercept*



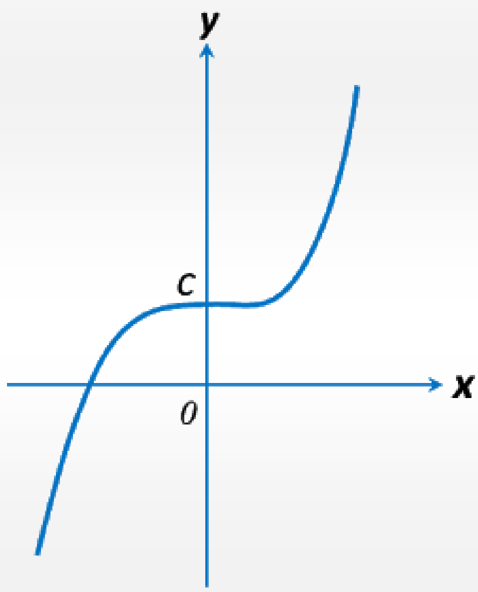
ii) y= $-ax^2 + bx + c, a < 0$ *and c is y intercept*



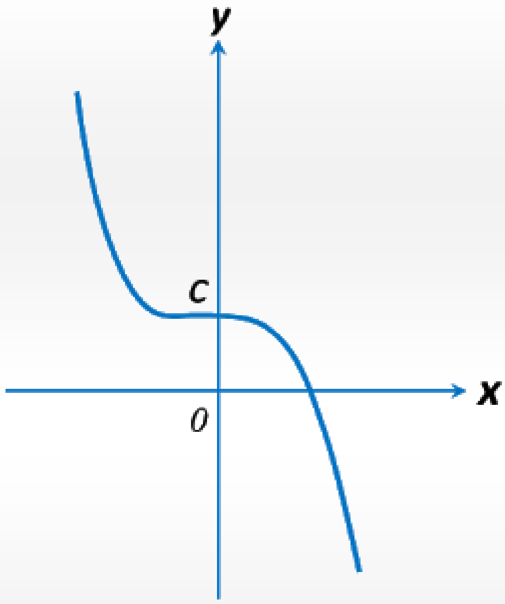
C. Cubic Function

A two-dimensional model of functions where x is raised to the third power is obtained by graphing cubic functions. Often written in y= $ax^3 + bx + c$. In some respects, charting cubic functions is comparable to graphing quadratic functions. The basic geometry of a cubic graph, in particular, can be used to aid in the creation of models for more intricate cubic functions. Highest power of the variable x is 3.

i) y= $ax^3 + bx + c, a > 0$ *and c in y intercept*



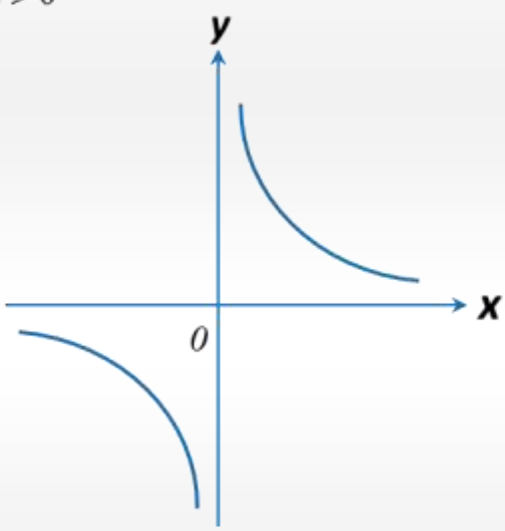
ii) y= $-ax^3 + bx + c, a < 0$ *and c in y intercept*



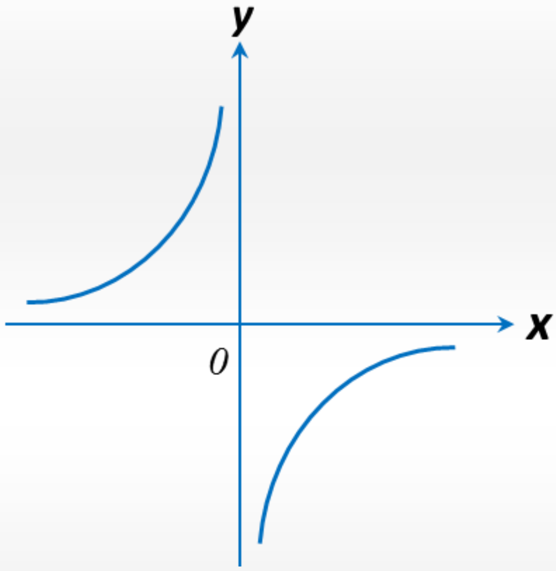
D. Reciprocal function

The form of reciprocal functions is $y = \frac{k}{x}$, where k is any real number. A symmetry line, as well as a horizontal and vertical asymptote, may be found in their graphs. To graph reciprocal functions, you must first learn how to graph the parent function, $y = \frac{k}{x}$. Highest power of the variable *x* is **−1**

$(i)\ y = \frac{a}{x}\ , a > 0$

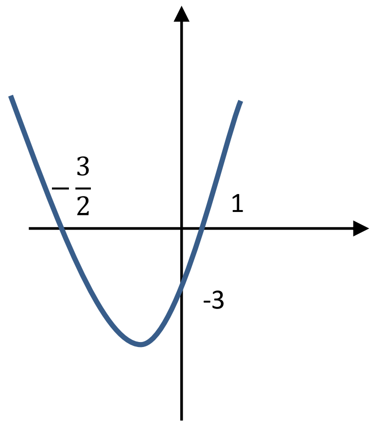


$(ii)\ y = \dfrac{a}{x}\ ,\ a < 0$



Example 1

Write the equation of this graph



Solution:

Since the root is x=$-\frac{3}{2}$ and x=1

$(2x+3)(x-1)=0$

$2x^2\ +x-3=0$

6.2 Sketching Graph Linear and Quadratic function

Find the y intercept and x intercept in linear and quadratic function.

To find the y intercept, $x = 0$
The value b (in linear function, $y = ax + b$) and the value of c (in quadratic function, $y = ax^2 + bx + c$)

To find the x intercept, $y = 0$
For quadratic, a>0 minimum point while a<0 maximum point

For Example:
Find the y intercept and x intercept of this equation:

$$y = -\frac{1}{2}x + 2$$

$y - intercept, x = 0$

$y = -\frac{1}{2}(0) + 2$

$y = 2 \; or \; value \; of \; b \; which \; is \; (4)$

$y \; intercept = (0,2)$

$y = -\frac{1}{2}x + 2$

$x - intercept, y = 0$

$0 = -\frac{1}{2}x + 2$

$-2 = -\frac{1}{2}x$

$-4 = -x$

$x = 4$

$x \; intercept = (4,0)$

A. Sketching the Linear Graph

Example 2

Sketch the following straight line graphs.

$$y = x + 4$$

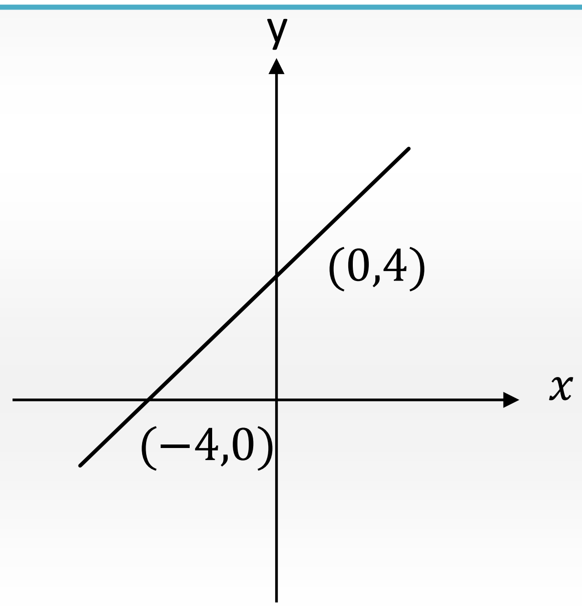
Solution:

$y - intercept, x = 0$

$y = (0) + 4$

$y = 4 \; or \; value \; of \; b \; which \; is \; (4)$

$y \; intercept = (0,4)$



$y = x + 4$

$x - intercept, y = 0$

$0 = x + 4$

$-4 = x$

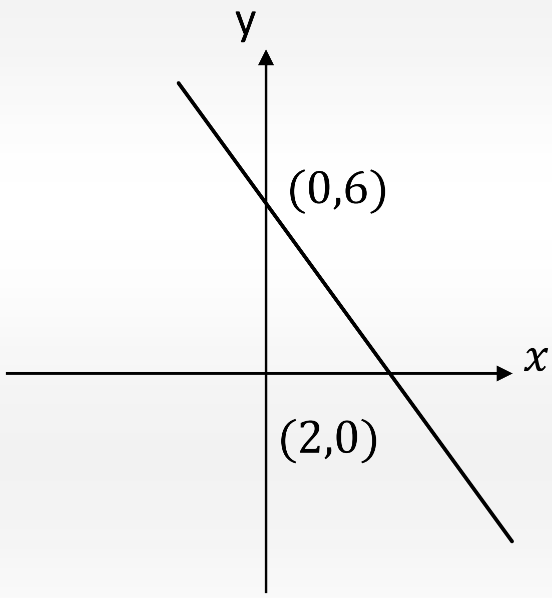
$x \; intercept = (-4,0)$

Example 3

Sketch the following straight line graphs.

$$y = -3x + 6$$

Solution:



$y \; - intercept, x \; = \; 0$

$y \; = \; -3(0) \; + \; 6$

$y \; = \; 6 \; or \; value \; of \; b \; which \; is \; (6)$

$y \; intercept = \; (0,6)$

$x \; - intercept, y \; = \; 0$

$0 \; = \; -3x + 6$

$\frac{-6}{-3} \; = \; x$

$x \; intercept = (2,0)$

B. Sketching the Quadratic Graph

Example 4

Sketch the following quadratic graphs

$$y = x^2 + x - 6$$

Solution:

$y - intercept, x = 0$

$y = (0)^2 + (0) - 6$

$y = -6$ *or value of c which is* (-6)

$(0, -6)$

$x - intercept, y = 0$

$0 = x^2 + x - 6$

By using a calculator $(a: 1, b: 1, c: -6)$

$y = (x + 3)(x - 2)$

$x = -3$ *and* $x = 2$

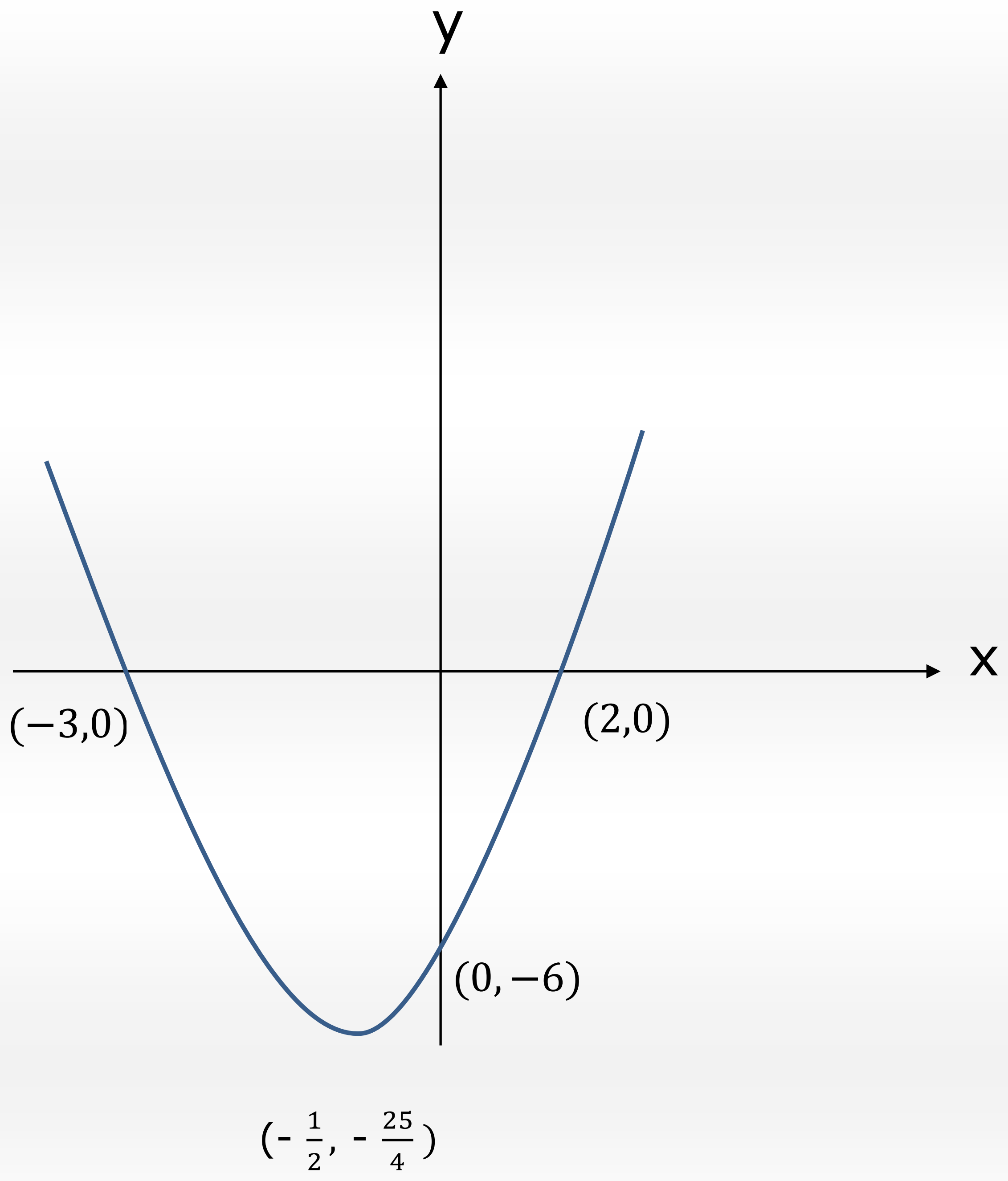
$(-3,0)\ (2,0)$

Minimum point:

x $= \frac{-(b)}{2a}$, y= substitute value of x into the function

x $= \frac{-(1)}{2(1)}$ $\quad y = (-\frac{1}{2})^2 + \left(-\frac{1}{2}\right) - 6$

x = - $\frac{1}{2}$ $\quad$ y= $-\frac{25}{4}$


y
x
$(-3,0)$
$(2,0)$
$(0,-6)$
$(-\frac{1}{2}, -\frac{25}{4})$

Example 5

Sketch the following quadratic graphs

$$y = -x^2 - x + 2$$

Solution:

$y - intercept, x = 0$

$y = -(0)^2 - (0) + 2$

$y = +2 \text{ or value of } c \text{ which is } (+2)$

$y \text{ intercept} = (0,2)$

$x - intercept, y = 0$

$0 = -x^2 - x + 2$

$By \text{ using a calculator } (a: -1, b: -1, c: +2)$

$y = (x - 1)(x + 2)$

$x = 1 \text{ and } x = -2$

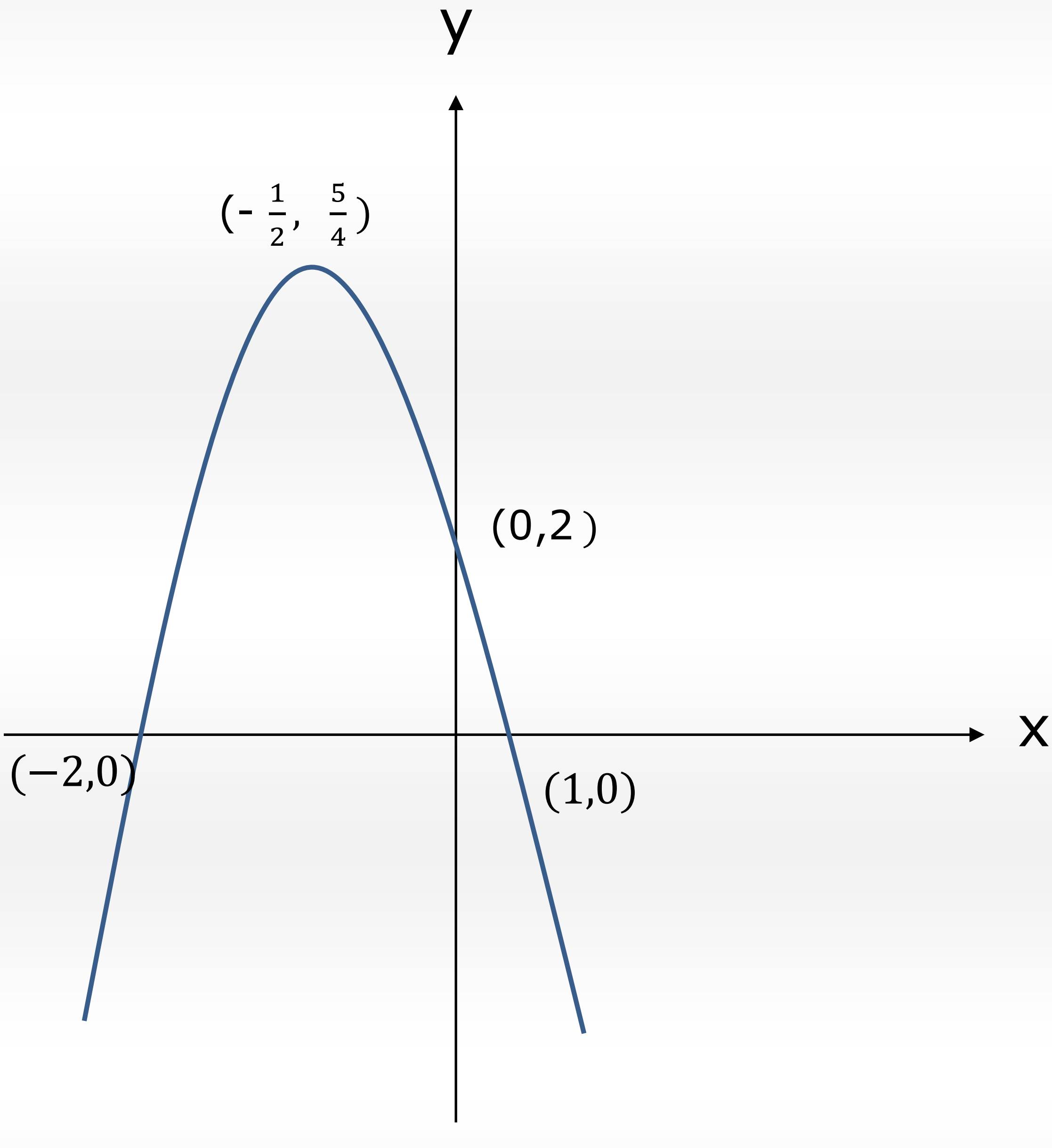
$x \text{ intercept} = (1,0)\ (-2,0)$

$Maximum \text{ point } a < 0:$

$x = \frac{-(b)}{2a}, y = \text{substitute value of } x \text{ into the function}$

$x = \frac{-(-1)}{2(-1)} \quad y = -(-\frac{1}{2})^2 + \left(-\frac{1}{2}\right) + 2$

$x = -\frac{1}{2} \quad y = \frac{5}{4}$


y
(- 1/2, 5/4)
(0,2)
x
(−2,0)
(1,0)

6.3 Find the value of x and y in the function

Example 6

$Fom\ the\ linear\ fuction\ y = 3x + 5,\ find\ the\ value\ of$

x		-1.4	1.1	-2.3
y	6.5	0.8	8.3	

Solution:

$y = 3x + 5$

$(6.5) = 3x + 5$

$6.5 - 5 = 3x$

$\frac{1.5}{3} = x$

$x = 0.5$

$y = 3x + 5$

$y = 3(-2.3) + 5$

$x = -6.9 + 5$

$x = 9$

6.4 Ploting a graph

Example 7

a) Complete table in the answer space for the equation $y=\frac{24}{x}$ by writing down the values of y when x=0.8 and x=5

x	0.5	0.8	1	2	2.4	4	5	6
y	48		24	12	10	6		4

b) By using a scale of 2 cm to 1 unit on x axis and 2 cm to 10 units on y axis, draw of graph $y=\frac{24}{x}\ for\ 0.5\leq x\leq 6$

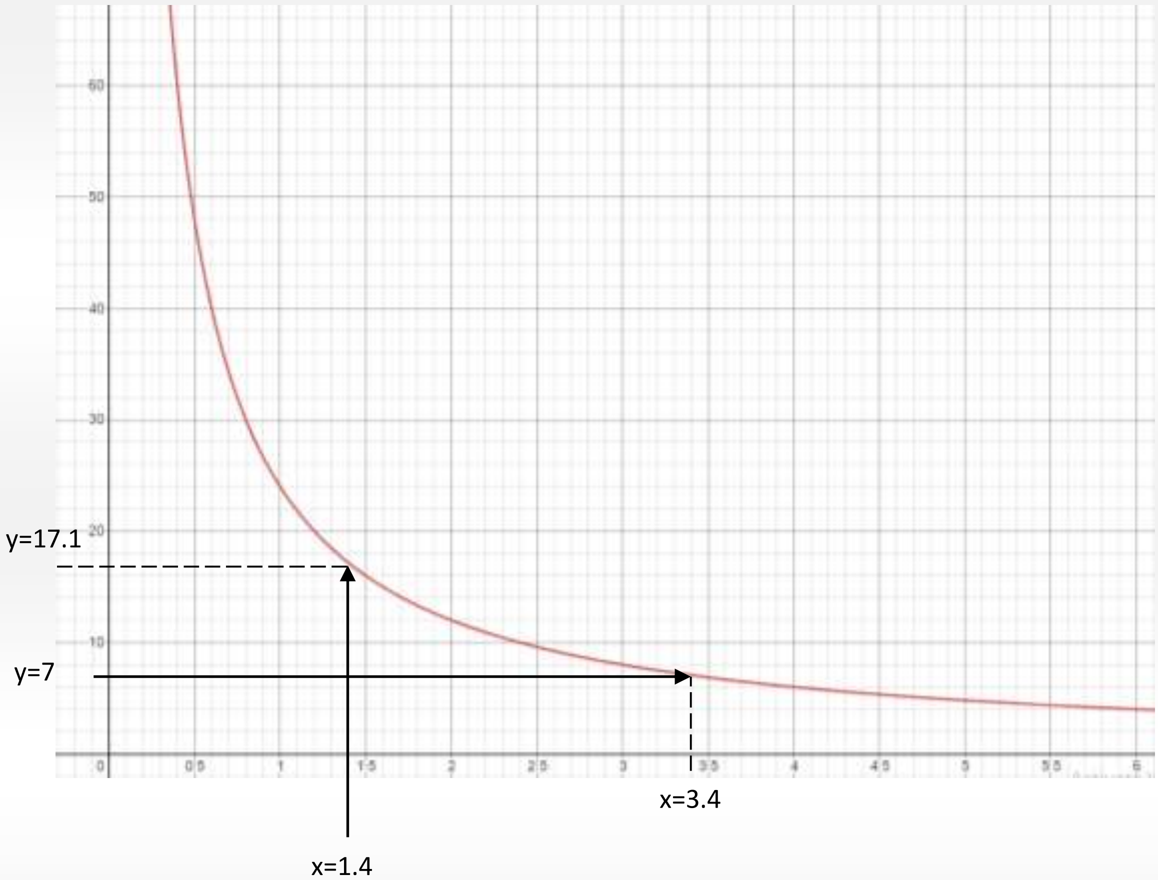
c) From the graph find the value of y when x=1.4 and the value of x when y=7

Solution:

the equation $y=\frac{24}{x}$

x	0.5	0.8	1	2	2.4	4	5	6
y	48	**30**	24	12	10	6	**4.8**	4

Graph of $y = \frac{24}{x}$



When x=1.4 , y=17.2

y=7 , x=3.4

Self Assessment

Exercise 6.1

a) Sketch the graph of following linear function $2y = 4x - 1$

b) sketch the graph of following quadratoc function $y = -x^2 + 16$

Exercise 6.2

a) Complete table in the answer space for the equation $y = -3x^2 - 5x + 16$ by writing down the values of y when x=-3 and x=1

x	-4	-3	-2	-1	0	1	2	3
y	-12		14	18	16		-6	-26

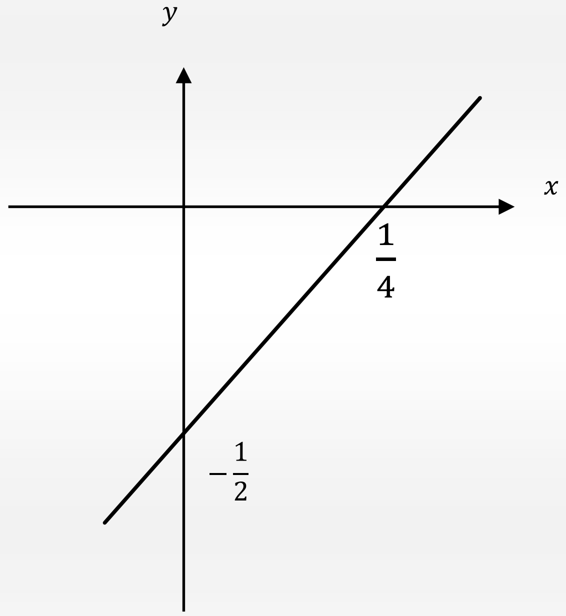
b) By using a scale of 2 cm to 1 unit on x axis and 2 cm to 10 units on y axis, draw of graph $y = -3x^2 - 5x + 16\ for - 4 \leq x \leq 3$

c) From the graph find the value of y when x=2.3 and the value of x when y=9

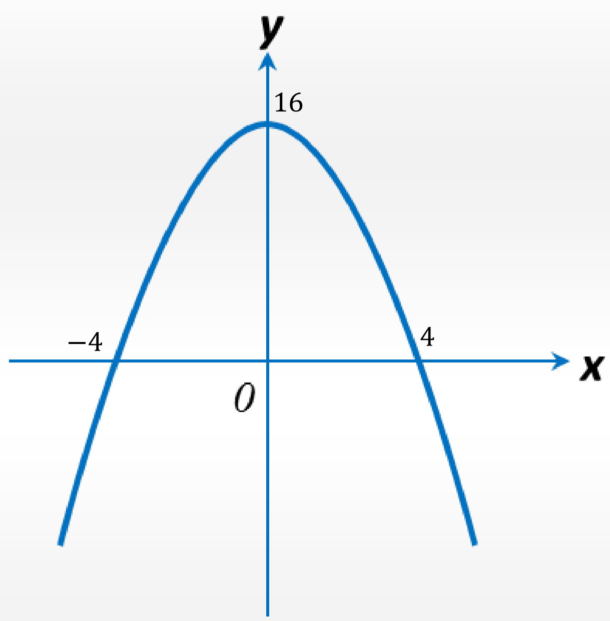
Answer

Exercise 6.1

a) Sketch the graph of following linear function $2y = 4x - 1$



b) sketch the graph of following quadratic function $y = -x^2 + 16$



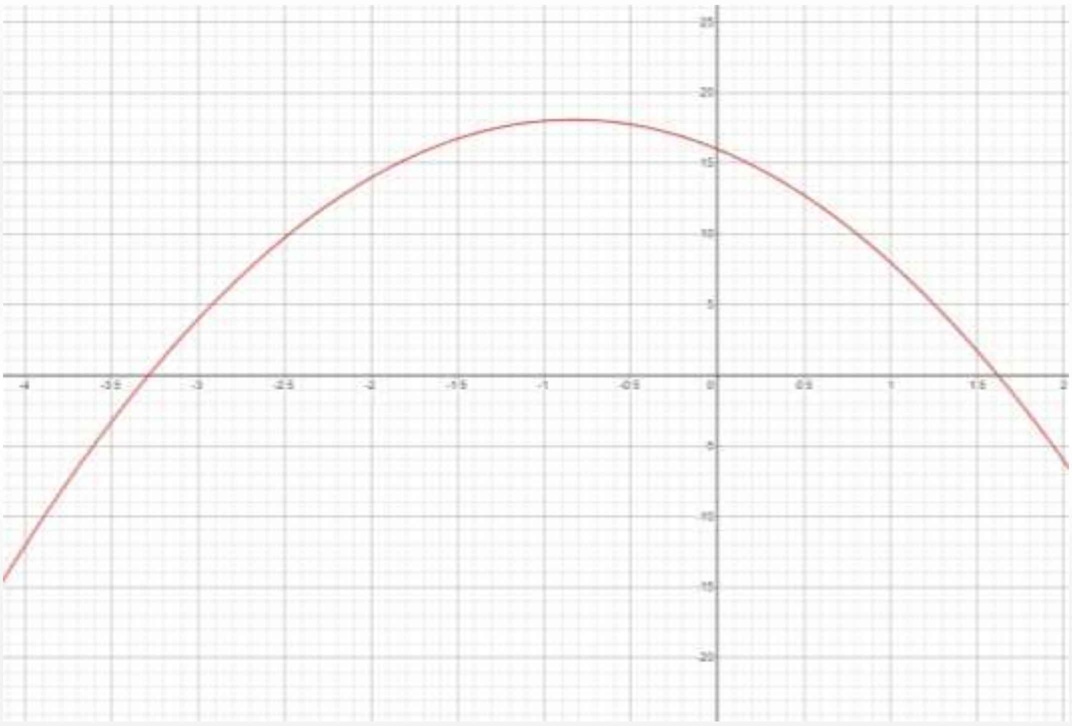
Exercise 6.2

a) Complete table in the answer space for the equation $y = -3x^2 - 5x + 16$ by writing down the values of y when x=-3 and x=1

x	-4	-3	-2	-1	0	1	2	3
y	-12	4	14	18	16	8	-6	-26

b) By using a scale of 2 cm to 1 unit on x axis and 2 cm to 10 units on y axis, draw of graph $y = -3x^2 - 5x + 16\ for - 4 \leq x \leq 3$

c) From the graph find the value of y when x=2.3 and the value of x when y=9



When x=2.3 , y=-11.5

y=9 , x=0.95,-2.57

KEY TERM

Graph | **Function**
Y intercept | **Plot**
X intercept | **Values of x and y**

SUMMARY

- A linear function, $y = ax + b$, where a, and b is constant.
- A quadratic function, $y = ax^2 + bx + c$ where a, b, and c are constant.
- A cubic function, $y = ax^3 + bx^2 + cx + d$ where a, b, c and d are constant
- A reciprocal function, $y = \frac{a}{x}$ when a is a constant and a ≠ 0
- x-intercept is when y=0 while y-intercept when x=0

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2. y intercept | x intercept | Find x and y intercepts. (2020). Cuemath. https://www.cuemath.com/learn/mathematics/functions-x-intercept-y-intercept/

Introduction to Logic

TOPIC 7

LEARNING OUTCOMES

By the end of topic, you should be able to:

1. Explain the statement, conjunction, disjunction, negation, conditional and Bi-conditional statements. Tautology, contradiction or contingency statements.
2. Construct truth table
3. Calculate Boolean Polynomial
4. Compute logically equivalent

INTRODUCTION

The study of the criteria used to evaluate inferences or arguments is referred to as logic.

An inference is a method of thinking in which a new belief is created based on or as a result of evidence or proof allegedly offered by prior beliefs.

An argument is a set of statements or propositions, some of which are intended to support or prove the validity of another.

Truth tables are a technique of looking at how the validity of statements (called propositions) changes when you combine them with a logical or, and, not, if...then , if and only if. Because propositions might be totally true or completely untrue, any truth table should display both of these options for each assertion.

7.1 Statement

Any sentences that is either true or false, but not both. It is a sentences that can be assigned the truth value (true) or the truth value (false) and not both. Statement cannot be question or instruction senteces. Statement also need to be clear without unknown.

The following are statements:

- My name is ali
- Today is Monday
- 3+6=9
- 2+2>3

The following are statements:

Example 2

- What is your name?
- No!
- 3x+2=4

Example 1

Defined wheter it is true or false statement:

a) a cube have 6 sides

b) 3+3 >4

c) Can you please take me to the town?

Solution:

a. False statement
b. True statement
c. Not a statement

7.2 Compound statement

Compound statement is made up of one or more simple statements that are linked together by connectives.

WHAT IS CONNECTIVES??

- Not
- And
- Or
- If.. Then.. (implies)
- If and only if

Example 1

Define the following compound statements with correct connective

a) It is raining and the sky is cloudy
b) If today is Sunday, Then I'm not going to work
c) $3x=3$ if and only if $x=1$
d) It is not true 3 is an even number.Can you please take me to the town?

Solution:

a. It is raining and the sky is cloudy
b. If today is Sunday, Then I'm not going to work
c. $3x=3$ if and only if $x=1$
d. It is not true 3 is an even number.

7.3 Negation, Conjunction, Disjunction, Conditional and Biconditional

Connectives	Notation	Name of Connectives
Not	~or ¬	Negation
And	^	Conjuction
Or	v	Disjunction
If..then	⇒	Conditional
If and only if	⇔	biconditional

Example 3

Fill up the tbale below with the correct name of connective and notation.

Compound Statement	Name of connective	Notation
If today is Friday, then i will go buy some stuff		
3+1 ≠ 4		
Mira go to school or Ahmad playing football.		
$x^2 = 9$ *if and only if* $x = 3$		

Solution:

Compound Statement	Name of connective	Notation
If today is Friday, then i will go buy some stuff	Conditional	→
3+1 ≠ 4	Negation	~
Mira go to school or Ahmad playing football.	Disjunction	V
$x^2 = 9$ *if and only if* $x = 3$	Biconditional	↔

7.4 Truth Table

Since there are only **four different combinations** of truth values for p and q, we can simply give table to describe the truth values of these compound statement for all possible combination.

One way to indicate truth values in a table would be use letters T for true and F for false

p	q
T	T
T	F
F	T
F	F

A. The truth table for (^) conjunction

- Two basic statements can be combined with the word "and" to form a conjunction of statement. The conjunction of *p* and *q* cam b written as:

$$p \wedge q$$

- Or conjunction statement for p and q can also written as: "p and q"

Example 4

If p represent the basic statement " 3 is an odd number" and q represent "3 can be divided with 2 ".

a) Construct a table for Conjunction
b) State the statement of conjunction and the nature of conjunction statement p and q given.

Solution:

If p represent the basic statement " 3 is an odd number" and q represent "3 can be divided with 2 ".

Construct a table:

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

a) The **statement of conjunction** of p and q are; 3 is an odd number **and** 3 can be divided with 2 ".
b) The **nature of conjunction statement** between p and q is FALSE. Because statement p "3 is an odd number" is <u>true</u> but statement q 3 can be divided with 2 is <u>false</u>. Thus**, true ^ false = false**

B. The truth table for (v) disjunction

Two basic statement can be combined with the word "or" to form a disjunction of statement. The disjunction of *p* and *q* can be written as:

$$p \vee q$$

Or disjunction statement for p and q can also written as : "p or q"

Example 5

If p represent the basic statement " 3 is an odd number" and q represent "3 can be divided with 2 ".

a) Construct a table for disjunction
b) State the statement of conjunction and the nature of conjunction statement p and q given.

Solution:

If p represent the basic statement " 3 is an odd number" and q represent "3 can be divided with 2 ".

Construct a table:

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

c) The **statement of disjunction** of p and q are; 3 is an odd number **or** 3 can be divided with 2 ".

d) The **nature of disjunction statement** between p and q is TRUE. Because statement p "3 is an odd number" is <u>true</u> but statement q 3 can be divided with 2 is <u>false</u>. Thus**, true ^ false =True**

C. The truth table for (~) negation

- Simple statement can be denied with the word "not" to form negation statement.
- Negation statement for p can be written as "not p " or denoted as $\sim p$

Example 6

If p represent the basic statement " 3 is an odd number" and q represent "3 can be divided with 2 ".

a) Construct a table for negation
b) State the statement of negation p and q and the nature of negation statement p and q given.

Solution:

If p represent the basic statement " 3 is an odd number" and q represent "3 can be divided with 2 ".

Construct a table:

p	q	$\sim p$	$\sim q$
T	T	F	F
T	F	F	T
F	T	T	F
F	F	T	T

The **statement of negation** of $\boldsymbol{p}$ is "3 is not odd number" while the **statement of negation** of q is "3 cannot be divided with 2"

The **nature of negation statement** for p "3 is an odd number" is TRUE but for $\sim p$ "3 is not odd number" is FALSE

The **nature of negation statement** for q "3 can be divided with 2" is FALSE but for $\sim p$ "3 cannot be divided with 2" is TRUE

D. The truth table for (⇒,→) conditional

Conditional statements for p and q can be denoted by: $p \Rightarrow q$

The truth table for conditional implication $p \Rightarrow q$ means that the truth of p implies the truth of q. In other words: "if p then q". Conditional statement "if p the q" can also be expressed in several ways, such as:

a) p if then q
b) p only if q
c) p sufficient for q

Example 7

If p represent the basic statement " 3 is an odd number" and q represent "3 can be divided with 2 ".

a) Construct a table for conditional
b) State the statement of conditional and the nature of conditional statement p and q given.

Solution:

If p represent the basic statement " 3 is an odd number" and q represent "3 can be divided with 2 ".

Construct a table:

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

The **statement of conditional** of p and q are; if 3 is an odd number **then** 3 can be divided with 2 ".

The **nature of conditional statement** between is False. Because statement p "3 is an odd number" is true and statement " 3 can be divided with 2" is FALSE. In other word, if p is true while p is false then $p \Rightarrow q$ is false. Thus**,**

True⇒False = False

E. The truth table for (↔) biconditional

Bi-conditional statements for p and q can be denoted by: $p \Leftrightarrow q$

The truth table for bi-conditional implication $p \Leftrightarrow q$ means that the truth of p implies the truth of q. In other words: " p if and only if q"

Example 8

If p represent the basic statement " 5 is an even number" and q represent "5 can be divided with 2 ".

c) Construct a table for Biconditional
d) State the statement of Biconditional and the nature of biconditional statement p and q given.

Solution:

If p represent the basic statement " 5 is an even number" and q represent "5 can be divided with 2 ".

Construct a table:

p	q	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

The **statement of biconditional** of p and q are; 5 is an even number if and only if 5 can be divided with 2 ".

The **nature of biconditional statement** between is True. Because statement p "5 is an even number" is false and statement " 5 can be divided with 2" is TRUE. In other word, if p is true while p is false then $p \Leftrightarrow q$ is true. Thus, **False ⇔False =True**

7.5 Boolean Polynomials

Is an expression built from the merger statement p,q,r.. By using connectives ^,v,~, ⇔ and ⇒.

Example 9

Let statement p represent "1+1=4" and q represent "1 year is 13 months".

Translate in compound statements for this ~p^(p ⇒q)

Solution:

Let statement p represent "1+1=4" and q represent "1 year is 13 months".

Translate in compound statements for this ~p^(p ⇒q)

~p^(p ⇒q)="1+1≠ 4, and if 1+1=4 the 1 year is 13 months".

Statement "1+1≠ 4" is true

Statement "1+1= 4" is false

Statement "1 year is 13 months" is false

"1+1= 4" ⇒ "1 year is 13 months"

F ⇒ F is True

The **conjunction statement** " true ^ true" is true, then T ⇒ T is true.

Boolean statement "1+1≠ 4, and if 1+1=4 the 1 year is 13 months" is true

7.6 Tautology and Contradiction

A compound statement that is always true called, Tautology (result all TRUE). Also known as **logically true.**

A compound statement that is always false called, Contradiction (result all FALSE). Also known as **logically false.**

A. Tautology

Example 10

Show that compound statement $(p \vee q) \vee \sim(p \wedge q)$ is a tautology

Solution:

p	q	$(p \vee q)$	$(p \wedge q)$	$\sim(p \wedge q)$	$(p \vee q) \vee \sim(p \wedge q)$
T	T	T	T	F	T
T	F	T	F	T	T
F	T	T	F	T	T
F	F	F	F	T	T

B. Contradiction

Example 11

Show that compound statement $\sim(p \vee \sim(p \wedge q))$ is a contradiction.

Solution:

p	q	$(p \wedge q)$	$\sim(p \wedge q)$	$(p \vee \sim(p \wedge q)$	$\sim(p \vee \sim(p \wedge q)$
T	T	T	F	T	F
T	F	F	T	T	F
F	T	F	T	T	F
F	F	F	T	T	F

7.7 Logically Equivalence

If two compound statements have the same truth value for all possible truth values for the variables, they are considered logically comparable by writing $p \equiv q$

Show that compound statement $(p \wedge q) \vee (p \wedge \sim q) \equiv p$ is a logically equivalence

p	q	$p \wedge q$	$\sim q$	$p \wedge \sim q$	$(p \wedge q) \vee (p \wedge \sim q)$	p
T	**T**	**T**	**F**	**F**	**T**	**T**
T	**F**	**F**	**T**	**T**	**T**	**T**
F	**T**	**F**	**F**	**F**	**F**	**F**
F	**F**	**F**	**T**	**F**	**T**	**F**

Self Assessment

Exercise 7.1

Construct the truth table for each of the following to prove logically equivalence.

1. $p\vee (q\vee (p\wedge\sim p)) \equiv (p\vee q)$
2. $p \rightarrow (p\vee q) \equiv (p^{q}) \rightarrow p$

Construct the truth tbale for each of the following to prove tautology.

1. $(\sim p\wedge(p\vee q)) \rightarrow q$
2. $((p \leftrightarrow q)\wedge p) \rightarrow q$

Construct the truth tbale for each of the following to prove contradiction.

1. $\sim((p \rightarrow (q\wedge\sim q)) \rightarrow \sim p)$

Answer

Exercise 7.1

Construct the truth table for each of the following to prove logically equivalence.

1. $p \vee (q \vee (p \wedge \sim p)) \equiv (p \vee q)$

p	q	$\sim p$	$p \wedge \sim p$	$q \vee (p \wedge \sim p$	$p \vee (q \vee (p \wedge \sim p))$		$(p \vee q)$
T	**T**	**F**	**F**	**T**	**T**		**T**
T	**F**	**F**	**F**	**F**	**T**	$\equiv$	**T**
F	**T**	**T**	**F**	**T**	**T**		**T**
F	**F**	**T**	**F**	**F**	**F**		**F**

2. $p \rightarrow (p \vee q) \equiv (p \wedge q) \rightarrow p$

p	q	$(p \vee q)$	$p \rightarrow (p \vee q)$		$(p \wedge q)$	$(p \wedge q) \rightarrow p$
T	**T**	**T**	**T**	$\equiv$	**T**	**T**
T	**F**	**T**	**T**		**F**	**T**
F	**T**	**T**	**T**		**F**	**T**
F	**F**	**F**	**T**		**F**	**T**

Construct the truth tbale for each of the following to prove tautology.

1. $(\sim(p \vee q) \rightarrow (\sim p \wedge \sim q)$

p	q	$(p \vee q)$	$\sim(p \vee q)$	$\sim p$	$\sim q$	$(\sim p \wedge \sim q)$	$(\sim(p \vee q) \rightarrow (\sim p \wedge \sim q)$
T	**T**	**T**	**F**	**F**	**F**	**F**	**T**
T	**F**	**T**	**F**	**F**	**T**	**F**	**T**
F	**T**	**T**	**F**	**T**	**F**	**F**	**T**
F	**F**	**F**	**T**	**T**	**T**	**T**	**T**

2. $((p \leftrightarrow q) \wedge p) \rightarrow q$

p	q	$p \leftrightarrow q$	$((p \leftrightarrow q) \wedge p$	q	$((p \leftrightarrow q) \wedge p) \rightarrow q$
T	**T**	**T**	**T**	**T**	**T**
T	**F**	**F**	**F**	**F**	**T**
F	**T**	**F**	**F**	**T**	**T**
F	**F**	**T**	**F**	**F**	**T**

Construct the truth tbale for each of the following to prove contradiction.

1. $\sim((p \rightarrow (q \wedge \sim q)) \rightarrow \sim p)$

p	q	$\sim q$	$(q \wedge \sim q)$	$p \rightarrow (q \wedge \sim q)$	$(\sim p)$	$(p \rightarrow (q \wedge \sim q)) \rightarrow \sim p$	$\sim((p \rightarrow (q \wedge \sim q)) \rightarrow \sim p)$
T	**T**	**F**	**F**	**F**	**F**	**T**	**F**
T	**F**	**T**	**F**	**F**	**F**	**T**	**F**
F	**T**	**F**	**F**	**T**	**T**	**T**	**F**
F	**F**	**T**	**F**	**T**	**T**	**T**	**F**

KEY TERM

Statement	**Conjunction**
Negation	**Conditional**
Disjunction	**Biconditional**
Tautology	**Contradiction**

SUMMARY

Connective	Name of connective	Notation
Not	Negation	~
And	Conjunction	^
Or	Disjunction	V
If ...then	Conditional	→
If and only if	Bi conditional	↔

- Truth Table contain True and False combination
- The result of Tautology is all true, while contradiction the results are all false.

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